S.A.-1827

SOLUTIONS

OF

THE MORE DIFFICULT EQUATIONS

CONTAINED

IN THE FOURTH EDITION

OF

Dr. BLAND'S ALGEBRAICAL PROBLEMS.

BY

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Pa. Jube SOLVI, obsecro.
Si. Age, fiat. Pa. At matura. Si. Eo intrò.
Pa. O faustum, et felicem, hunc diem!—Ter.

LONDON:
PRINTED FOR J. MAWMAN, LUDGATE-STREET.

MDCCCXXVII.

LONDON
Printed by W. CLOWES,
Stamford-street.

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In publishing the following Solutions, the Author has no intention of producing indolence in the learner. He has found that the beginner, who has no tutor to consult, and is compelled to depend upon his own resources, is often discouraged at the very commencement of his attempts. In fact, the solution of the majority of the following Equations consists in some trick—when that is discovered, all is finished. *Ce n'est que le premier pas qui coute.* But to take this first step is the very difficulty—and it is the object of the following pages to remove it.

The Author has given Solutions only for the last few of the Simple Equations involving one unknown Quantity; they appear to be the only ones requiring explanation.

The Simple Equations involving two unknown Quantities are omitted altogether—they may all be solved by application of the ordinary rules.

The Pure Quadratics are all solved. Solutions of the Adfected Quadratics involving one unknown Quantity

will be found for all that are really difficult; and similarly for those involving two unknown quantities.

The Reader will thus perceive that it has been the Author's aim to produce a compendious and useful manual. He might have swelled it out by giving Solutions to all the Equations—but this was far from his design.

Should this tract be received with approbation, it is the Author's intention to prepare Solutions of the more difficult Problems contained in the same work.

SOLUTIONS

OF

SIMPLE AND QUADRATIC EQUATIONS.

Solutions of Simple Equations involving one unknown Quantity.
Bland, p. 299.

46. Given $\sqrt[3]{10x + 35} - 1 = 4$, to find the value of x.

$$\text{Transposing } \sqrt[3]{10x + 35} = 5,$$
$$\text{and cubing each side, } 10x + 35 = 125;$$
$$\therefore\ 10x = 90,$$
$$\text{and } \quad x = 9.$$

47. Given $\sqrt[5]{9x - 4} + 6 = 8$, to find the value of x.

$$\text{By transposition } \sqrt[5]{9x - 4} = 2,$$

and involving each side to the fifth power,

$$9x - 4 = 32;$$
$$\therefore\ 9x = 36,$$
$$\text{and } x = 4.$$

48. Given $\sqrt{x + 16} = 2 + \sqrt{x}$, to find the value of x.

Squaring both sides, $x + 16 = 4 + 4\sqrt{x} + x$;

$$\therefore\ 4\sqrt{x} = 12,$$
$$\sqrt{x} = 3,$$
$$\text{and } x = 9.$$

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49. Given $\sqrt{x - 32} = 16 - \sqrt{x}$, to find the value of x.

Squaring both sides, $x - 32 = 256 - 32\sqrt{x} + x$;

$$\therefore\ 32\sqrt{x} = 288,$$
$$\sqrt{x} = 9,$$
$$\text{and } x = 81.$$

50. Given $\sqrt{4x + 21} = 2\sqrt{x} + 1$, to find the value of x.

Squaring each side, $4x + 21 = 4x + 4\sqrt{x} + 1$;

$$\therefore\ 4\sqrt{x} = 20,$$
$$\sqrt{x} = 5,$$
$$\text{and } x = 25.$$

51. Given $a.\sqrt[3]{bx - c} = d.\sqrt[3]{ex + fx - g}$, to find the value of x.

Cubing each side, $a^3(bx - c) = d^3(ex + fx - g)$,

or $a^3bx - a^3c = d^3ex + d^3fx - d^3g$;

by transposition, $a^3bx - d^3ex - d^3fx = a^3c - d^3g$,

or $x(a^3b - d^3.(e + f)) = a^3c - d^3g$;

$$\therefore\ x = \frac{a^3c - d^3g}{a^3b - d^3.(e + f)}.$$

52. Given $\sqrt[3]{a^2 + c} = \sqrt[4]{\dfrac{a^2 + c}{d.(x + b)}}$, to find the value of x.

Involving each side to the 12th power,

$$(a^2 + c)^4 = \left(\frac{a^2 + c}{d.(x + b)}\right)^3,$$

$$\text{or } (d.\overline{x+b})^3 = \frac{1}{a^2 + c};$$

Extracting the cube-root on both sides,

$$d.\overline{x+b} = \frac{1}{\sqrt[3]{a^2+c}};$$

$$\therefore x+b = \frac{1}{d.\sqrt[3]{a^2+c}},$$

$$\text{and } x = \frac{1}{d.\sqrt[3]{a^2+c}} - b.$$

53. Given $\sqrt[m]{a+x} = \sqrt[2m]{x^2+5ax+b^2}$, to find the value of x.

Raising both sides to the $2m^{\text{th}}$ power,

$$a^2 + 2ax + x^2 = x^2 + 5ax + b^2,$$

$$\text{or } \quad 3ax = a^2 - b^2;$$

$$\therefore x = \frac{a^2 - b^2}{3a}.$$

54. Given $a + b.\sqrt[m]{x+d} = c$.

$$\text{Now } \sqrt[m]{x+d} = \frac{c-a}{b};$$

$$\therefore x + d = \left(\frac{c-a}{b}\right)^m,$$

$$\text{and } x = \left(\frac{c-a}{b}\right)^m - d.$$

55. Given $\dfrac{\sqrt{9x}-4}{\sqrt{x}+2} = \dfrac{15+\sqrt{9x}}{\sqrt{x}+40}$, to find the value of x.

Clearing of fractions, we have,

$$3x + 116\sqrt{x} - 160 = 3x + 21\sqrt{x} + 30;$$

$$\text{then } 95\sqrt{x} = 190,$$

$$\sqrt{x} = 2,$$

$$\text{and } x = 4.$$

56. Given $\frac{\sqrt{x}+\sqrt{b}}{\sqrt{x}-\sqrt{b}} = \frac{a}{b}$, to find the value of x.

This may be solved by two methods—1st, by the common rules of solution; and 2ndly, by the assistance of proportion.

$$\text{1st, then, } b\sqrt{x} + b\sqrt{b} = a\sqrt{x} - a\sqrt{b},$$

$$\text{or } a\sqrt{x} - b\sqrt{x} = a\sqrt{b} + b\sqrt{b};$$

$$\therefore \sqrt{x} = \sqrt{b}\left(\frac{a+b}{a-b}\right),$$

$$\text{and } x = b . \overline{\frac{a+b}{a-b}}\Big|^2 .$$

2ndly, more neatly thus:

$$\sqrt{x} + \sqrt{b} : \sqrt{x} - \sqrt{b} :: a : b,$$

$$\text{or } \sqrt{x} : \sqrt{b} :: a + b : a - b, \text{ (\textit{Wood's Alg.}, 182.)}$$

$$\therefore \sqrt{x} = \sqrt{b}.\left(\frac{a+b}{a-b}\right),$$

$$\text{and } x = b.\overline{\frac{a+b}{a-b}}\Big|^2 .$$

57. Given $\frac{\sqrt{6x}-2}{\sqrt{6x}+2} = \frac{4\sqrt{6x}-9}{4\sqrt{6x}+6}$, to find the value of x.

$$\text{Now } \sqrt{6x} + 2 : \sqrt{6x} - 2 :: 4\sqrt{6x} + 6 : 4\sqrt{6x} - 9,$$

$$\text{or } \sqrt{6x} : 2 :: 8\sqrt{6x} - 3 : 15 \quad (\textit{Alg. } 182);$$

$$\therefore 15\sqrt{6x} = 16\sqrt{6x} - 6,$$

$$\sqrt{6x} = 6,$$

$$6x = 36,$$

$$\text{and } x = 6.$$

58. Given $\frac{5x-9}{\sqrt{5x}+3} - 1 = \frac{\sqrt{5x}-3}{2}$, to find the value of x.

$$\text{Then } (\sqrt{5x}-3) - 1 = \frac{\sqrt{5x}-3}{2},$$

$$\text{or } \frac{\sqrt{5x}-3}{2} = 1;$$

$$\therefore \sqrt{5x} - 3 = 2,$$

$$\sqrt{5x} = 5,$$

$$5x = 25,$$

$$\text{and } x = 5.$$

59. Given $\sqrt{1+x\sqrt{x^2+12}} = 1+x$, to find the value of x, squaring both sides of the equation.

$$1 + x\sqrt{x^2+12} = 1 + 2x + x^2,$$

$$\text{and } \sqrt{x^2+12} = x + 2 \qquad \text{or } \underline{x=0}$$

then squaring each side, $x^2 + 12 = x^2 + 4x + 4$.

$$\therefore 4x = 8$$

$$\text{and } x = 2$$

60. Given $\frac{ax}{b}.\sqrt{c^2x^2+d^2} + \frac{acx^2}{b} = ex$, to find the value of x.

Dividing both sides by $\frac{x}{b}$,

$$a\sqrt{c^2x^2+d^2} + acx = eb \qquad \text{or } x=0$$

by transposition $a\sqrt{c^2x^2+d^2} = eb - acx$,

squaring both sides, $a^2c^2x^2 + a^2d^2 = e^2b^2 - 2abcex + a^2c^2x^2$.

$$\therefore 2abcex = b^2e^2 - a^2d^2$$

$$\text{and } x = \frac{b^2e^2 - a^2d^2}{2abce}$$

61. Given $\sqrt{x} + \sqrt{x-9} = \frac{36}{\sqrt{x-9}}$, to find the value of x.

Clearing of fractions,

$$\sqrt{x^2-9x} + x - 9 = 36$$

$$\text{and } \sqrt{x^2-9x} = 45 - x$$

$$\text{squaring both sides, } x^2 - 9x = 2025 - 90x + x^2.$$

$$\therefore 81x = 2025$$

$$\text{and } x = 25.$$

Solutions of Pure Quadratics and others, &c.

1. Given $3x^2 - 4 = 28 + x^2$, to find the values of x.

$$\therefore 2x^2 = 32$$

$$x^2 = 16$$

$$\text{and } x = \pm\ 4$$

2. Given $\left.\begin{array}{l} x + y : y :: 3 : 1 \\ \text{and } xy = 18 \end{array}\right\}$ to find the values of x and y.

From the first equation, $x : y :: 2 : 1$ (*Alg.* 180.)

$$\therefore x = 2y$$

$$\text{then by substitution } 2y^2 = 18$$

$$\therefore y^2 = 9$$

$$\text{and } y = \pm 3$$

3. Given $\left.\begin{array}{l} x - y : y :: 4 : 5 \\ \text{and } x^2 + 4y^2 = 181 \end{array}\right\}$ to find the values of x and y.

From the first equation, $x : y :: 9 : 5$. (*Alg.* 179.)

$$\text{and } x = \frac{9y}{5}$$

substituting this value in the second equation,

$$\frac{81y^2}{25} + 4y^2 = 181$$

$$\therefore 81y^2 + 100y^2 = 25 \times 181$$
$$y^2 = 25$$
$$\text{and } y = \pm 5$$
$$\text{and } x = \frac{9y}{5} = \pm 9.$$

4. Given $\left.\begin{array}{l} x + y : x - y :: a : b \\ \text{and } xy = c^2 \end{array}\right\}$ to find the values of x and y.

From the first equation, $x : y :: a + b : a - b$. (*Alg.* 182.)

$$\text{and } x = y.\frac{a+b}{a-b}$$

Then substituting this value in the second equation,

$$y^2.\frac{a+b}{a-b} = c^2$$
$$\text{and } y^2 = c^2.\frac{a-b}{a+b},$$
$$y = \pm c\sqrt{\frac{a-b}{a+b}},$$
$$\text{and } x = y.\frac{a+b}{a-b} = \pm c.\sqrt{\frac{a+b}{a-b}}.$$

5. Given $\left.\begin{array}{l} x^2 + y^2 : x^2 - y^2 :: 17 : 8 \\ \text{and } xy^2 = 45 \end{array}\right\}$ to find the values of x and y.

From the first equation, $x^2 : y^2 :: 25 : 9$. (*Alg.* 182.)

and $x : y :: 5 : 3$. (*Alg.* 188.)

$$\therefore x = \frac{5y}{3},$$

substituting this value in the second equation,

$$\frac{5y^3}{3} = 45;$$
$$\therefore y^3 = 27,$$
$$\text{and } y = 3;$$
$$\therefore x = \frac{5y}{3} = 5.$$

6. Given $\left.\begin{aligned} x^2 - xy &= 54 \\ \text{and } xy - y^2 &= 18 \end{aligned}\right\}$ to find the values of x and y.

by subtraction, $x^2 - 2xy + y^2 = 36$,

extracting the square root $x - y = \pm 6$.

Then from the first equation, $\therefore x.\overline{x-y} = 54$,

$$\text{and } x = \frac{54}{\pm 6} = \pm 9;$$

and from the second equation, $y.\overline{x-y} = 18$;

$$\therefore y = \frac{18}{\pm 6} = \pm 3.$$

7. Given $\left.\begin{aligned} x + y : x^2 - y^2 :: 1 : 4 \\ \text{and } xy = 21 \end{aligned}\right\}$ to find the values of x and y.

Dividing the two first terms of the proportion by x and y.

$$1 : x - y :: 1 : 4;$$

$$\therefore x - y = 4,$$

squaring both sides, $x^2 - 2xy + y^2 = 16$,

but $4xy = 84$

by addition, $x^2 + 2xy + y^2 = 100$;

$\therefore x + y = \pm 10$,

but $x - y = 4$;

$$\therefore x = 7 \text{ or } -3;$$

$$\text{and } y = 3 \text{ or } -7.$$

8. Given $\left.\begin{aligned} ax^2 + bxy &= c^3 \\ \text{and } x - y : x &:: m : n \end{aligned}\right\}$ to find the values of x and y.

From the second equation, $y : x :: n - m : n$;

$$\therefore y = x . \frac{n - m}{n},$$

Substituting this value in the first equation,

$$ax^2 + bx^2 . \frac{n-m}{n} = c^3;$$

$$\therefore x^2 \left(\frac{na+nb-mb}{n}\right) = c^2;$$

$$\text{and } x^2 = \frac{nc^2}{na+nb-mb};$$

$$\therefore x = \pm c \,.\, \sqrt{\frac{nc}{na+nb-mb}}.$$

$$\text{and } y = \frac{x.\overline{n-m}}{n} = \pm \frac{n-m}{n} \,.\, c \,.\, \sqrt{\frac{nc}{na+nb-mb}}.$$

9. Given $x^3+y^3 : x^3-y^3 :: 559 : 127$ and $x^2y = 294$ } to find the values of x and y.

From the first equation, $2x^3 : 2y^3 :: 686 : 432$. (*Alg.* 182.)

$$\text{or } x^3 : y^3 :: 343 : 216;$$

$$\text{or } x : y :: 7 : 6. \quad (\textit{Alg.}\ 188.);$$

$$\text{and } y = \frac{6x}{7}.$$

$$\text{Now } \frac{6x^3}{7} = 294;$$

$$\text{or } \frac{x^3}{7} = 49;$$

$$\text{then } x^3 = 343;$$

$$\therefore x = 7,$$

$$\text{and } y = \frac{6x}{7} = 6.$$

10. Given $x^2 - xy : xy - y^2 :: 3 : 7$ and $xy^2 = 147$ } to find the values of x and y.

Dividing the proportion by $x-y$ we have,

$$x : y :: 3 : 7;$$

$$\therefore x = \frac{3y}{7}.$$

Substituting this value in the second equation,

$$\frac{3y^3}{7} = 147;$$

$$\therefore \frac{y^3}{7} = 49;$$

$$y^3 = 343,$$

$$\text{and } y = 7;$$

$$\therefore x = \frac{3y}{7} = 3.$$

11. Given $\left.\begin{array}{r}\sqrt{x}+\sqrt{y} : \sqrt{x}-\sqrt{y} :: 4 : 1\\ \text{and } x-y=16\end{array}\right\}$ to find the values of x and y.

From the first equation, $\sqrt{x} : \sqrt{y} :: 5 : 3$. (*Alg.* 182.)

$$\text{and } \sqrt{x} = \frac{5\sqrt{y}}{3};$$

$$\therefore x = \frac{25y}{9}.$$

Substituting this value in the second equation,

$$\frac{25y}{9} - y = 16,$$

$$25y - 9y = 144;$$

$$\therefore y = \frac{144}{16} = 9,$$

$$\text{and } x = \frac{25y}{9} = 25.$$

12. Given $\left.\begin{array}{r}\sqrt[4]{x}-\sqrt[4]{y}=3\\ \text{and } \sqrt[4]{x}+\sqrt[4]{y}=7\end{array}\right\}$ to find the values of x and y.

$$\therefore \sqrt[4]{x} = 5;$$

$$\text{and } x = 625.$$

$$\text{Again, } \sqrt[4]{y} = 2,$$
$$y = 16.$$

13. Given $\left.\begin{array}{l} x - y : \sqrt{x} - \sqrt{y} :: 8 : 1 \\ \text{and } \sqrt{xy} = 15 \end{array}\right\}$ to find the values of x and y.

Dividing the first equation by $\sqrt{x} - \sqrt{y}$;

$$\therefore \sqrt{x} + \sqrt{y} : 1 :: 8 : 1,$$
$$\text{and } \sqrt{x} + \sqrt{y} = 8;$$

squaring each side $x + 2\sqrt{xy} + y = 64$;

$$\text{but } 4\sqrt{xy} \quad = 60;$$

$$\therefore x - 2\sqrt{xy} + y = 4,$$
$$\text{and } \sqrt{x} - \sqrt{y} = \pm 2;$$
$$\text{but } \sqrt{x} + \sqrt{y} = 8;$$

$$\therefore \sqrt{x} = 5 \text{ or } 3,$$
$$x = 25 \text{ or } 9;$$
$$\text{and } \sqrt{y} = 3 \text{ or } 5;$$
$$\text{and } y = 9 \text{ or } 25.$$

14. Given $\left.\begin{array}{l} x^3 - y^3 : x^2y - xy^2 :: 7 : 2 \\ \text{and } x + y = 6 \end{array}\right\}$ to find the values of x and y.

From the first equation, $x^3 - y^3 : 3x^2y - 3xy^2 :: 7 : 6$;

and $x^3 - 3x^2y + 3xy^2 - y^3 : x^2y - xy^2 :: 1 : 2$. (*Alg.* 180.)

$$\text{or } (x - y)^3 : xy.(x - y) :: 1 : 2;$$
$$\therefore (x - y)^2 : xy :: 1 : 2;$$
$$\text{and } x^2 - 2xy + y^2 : 4xy :: 1 : 8;$$
$$\therefore x^2 - 2xy + y^2 : x^2 + 2xy + y^2 :: 1 : 9,$$
$$x - y : x + y :: 1 : \pm 3;$$
$$\text{and } x - y : 6 :: 1 : \pm 3;$$

Substituting this value in the second equation,

$$\frac{3y^3}{7} = 147;$$

$$\therefore \frac{y^3}{7} = 49;$$

$$y^3 = 343,$$

$$\text{and } y = 7;$$

$$\therefore x = \frac{3y}{7} = 3.$$

11. Given $\left.\begin{array}{l}\sqrt{x}+\sqrt{y} : \sqrt{x}-\sqrt{y} :: 4 : 1 \\ \text{and } x - y = 16\end{array}\right\}$ to find the values of x and y.

From the first equation, $\sqrt{x} : \sqrt{y} :: 5 : 3$. (*Alg.* 182.)

$$\text{and } \sqrt{x} = \frac{5\sqrt{y}}{3};$$

$$\therefore x = \frac{25y}{9}.$$

Substituting this value in the second equation,

$$\frac{25y}{9} - y = 16,$$

$$25y - 9y = 144;$$

$$\therefore y = \frac{144}{16} = 9,$$

$$\text{and } x = \frac{25y}{9} = 25.$$

12. Given $\left.\begin{array}{l}\sqrt[4]{x}-\sqrt[4]{y} = 3 \\ \text{and } \sqrt[4]{x}+\sqrt[4]{y} = 7\end{array}\right\}$ to find the values of x and y.

$$\therefore \sqrt[4]{x} = 5;$$

$$\text{and } x = 625.$$

Again, $\sqrt[4]{y} = 2,$

$y = 16.$

13. Given $x - y : \sqrt{x} - \sqrt{y} :: 8 : 1$ and $\sqrt{xy} = 15$ } to find the values of x and y.

Dividing the first equation by $\sqrt{x} - \sqrt{y}$;

$\therefore \sqrt{x} + \sqrt{y} : 1 :: 8 : 1,$

and $\sqrt{x} + \sqrt{y} = 8;$

squaring each side $x + 2\sqrt{xy} + y = 64;$

but $4\sqrt{xy} \qquad = 60;$

$\therefore x - 2\sqrt{xy} + y = 4,$

and $\sqrt{x} - \sqrt{y} = \pm 2;$

but $\sqrt{x} + \sqrt{y} = 8;$

$\therefore \sqrt{x} = 5 \text{ or } 3,$

$x = 25 \text{ or } 9;$

and $\sqrt{y} = 3 \text{ or } 5;$

and $y = 9 \text{ or } 25.$

14. Given $x^3 - y^3 : x^2y - xy^2 :: 7 : 2$ and $x + y = 6$ } to find the values of x and y.

From the first equation, $x^3 - y^3 : 3x^2y - 3xy^2 :: 7 : 6;$

and $x^3 - 3x^2y + 3xy^2 - y^3 : x^2y - xy^2 :: 1 : 2.$ (*Alg.* 180.)

or $(x - y)^3 : xy.(x-y) :: 1 : 2;$

$\therefore (x - y)^2 : xy :: 1 : 2;$

and $x^2 - 2xy + y^2 : 4xy :: 1 : 8;$

$\therefore x^2 - 2xy + y^2 : x^2 + 2xy + y^2 :: 1 : 9,$

$x - y : x + y :: 1 : \pm 3;$

and $x - y : 6 :: 1 : \pm 3;$

$$\therefore x - y = \pm 2,$$
$$\text{and } x + y = 6.$$

$$\therefore x = 4 \text{ or } 2,$$
$$\text{and } y = 2 \text{ or } 4.$$

15. Given $\left.\begin{array}{r} \dfrac{1}{x} + \dfrac{1}{y} = \dfrac{1}{2} \\ \text{and } \dfrac{2}{xy} = \dfrac{1}{9} \end{array}\right\}$ to find the values of x and y.

From the first equation, $x + y = \dfrac{xy}{2}$;

$$\text{and } x + y = \frac{18}{2} = 9,$$
$$\therefore x^2 + 2xy + y^2 = 81,$$
$$\text{but } 4xy = 72.$$

by subtraction, $x^2 - 2xy + y^2 = 9$;

$$\therefore x - y = \pm 3,$$
$$\text{and } x + y = 9;$$

$$\therefore x = 6 \text{ or } 3;$$
$$\text{and } y = 3 \text{ or } 6.$$

16. Given $\left.\begin{array}{r} x^4 - y^4 = 369 \\ \text{and } x^2 - y^2 = 9 \end{array}\right\}$ to find the values of x and y.

Dividing the first equation by the second,

$$x^2 + y^2 = 41,$$
$$\text{but } x^2 - y^2 = 9.$$

$$\therefore x^2 = 25,$$
$$\text{and } x = \pm 5,$$
$$y^2 = 16;$$
$$\text{and } y = \pm 4.$$

17. Given $\left.\begin{array}{r} x^3 - y^3 = 56, \\ \text{and } x - y = \dfrac{16}{xy} \end{array}\right\}$ to find the values of x and y.

$$x^3 - y^3 = 56,$$

$$\text{and } 3x^2y - 3xy^2 = 48 \text{ (from the 2nd equation)};$$

$$\therefore \text{ by subtraction, } x^3 - 3x^2y + 3xy^2 - y^3 = 8,$$

and extracting the cube-root on both sides,

$$x - y = 2;$$

$$\therefore x^2 - 2xy + y^2 = 4,$$

$$\text{but } 4xy = 32;$$

$$\therefore x^2 + 2xy + y^2 = 36,$$

$$\text{and } x + y = \pm 6;$$

$$\text{but } x - y = 2.$$

$$\therefore x = 4 \text{ or } -2,$$

$$\text{and } y = 2 \text{ or } -4.$$

18. Given $\dfrac{1}{1-\sqrt{1-x^2}} - \dfrac{1}{1+\sqrt{1-x^2}} = \dfrac{\sqrt{3}}{x^2}$, to find the values of x.

Clearing the equation of fractions,

$$1+\sqrt{1-x^2}-1+\sqrt{1-x^2}=\left(\overline{1-\sqrt{1-x^2}}.\overline{1+\sqrt{1-x^2}}\right).\frac{\sqrt{3}}{x^2},$$

$$\text{or } 2\sqrt{1-x^2}=x^2.\frac{\sqrt{3}}{x^2},$$

$$\text{or } 2\sqrt{1-x^2} = \sqrt{3},$$

$$\text{or } \sqrt{1-x^2} = \frac{\sqrt{3}}{2};$$

$$\therefore 1 - x^2 = \frac{3}{4},$$

$$x^2 = \frac{1}{4},$$

$$\text{and } x = \pm\frac{1}{2}.$$

19. $\left.\begin{array}{l} x^2y + y^2 = 116 \\ \text{and } xy^{\frac{1}{2}} + y = 14 \end{array}\right\}$ to find the values of x and y.

Squaring the second equation, we have

$$\begin{aligned} x^2y + 2xy^{\frac{3}{2}} + y^2 &= 196 \\ \text{but } x^2y \qquad\quad + y^2 &= 116; \\ \hline \therefore\ 2xy^{\frac{3}{2}} &= 80, \\ \text{and } x^2y - 2xy^{\frac{3}{2}} + y^2 &= 36; \\ \therefore\ xy^{\frac{1}{2}} - y &= \pm 6, \\ \text{but } xy^{\frac{1}{2}} + y &= 14; \\ \hline \therefore\ y &= 4 \text{ or } 10, \\ \text{and } xy^{\frac{1}{2}} &= 10 \text{ or } 4; \\ \therefore\ x &= \frac{10}{y^{\frac{1}{2}}} \text{ or } \frac{4}{y^{\frac{1}{2}}}, \\ &= \frac{10}{2}, \text{ or } \frac{4}{\sqrt{10}}, \\ &= 5, \text{ or } \sqrt{\frac{16}{10}}, \\ &= 5, \text{ or } 2\sqrt{\frac{2}{5}}. \end{aligned}$$

20. Given $\left.\begin{array}{l} \sqrt[3]{x} + \sqrt[3]{y} = 6, \\ \text{and } x + y = 72, \end{array}\right\}$ to find the values of x and y.

Cubing the first equation, we have

$$\begin{aligned} x + 3x^{\frac{2}{3}}y^{\frac{1}{3}} + 3x^{\frac{1}{3}}y^{\frac{2}{3}} + y &= 216, \\ \text{but } x \qquad\qquad\qquad + y &= 72; \\ \hline \therefore\ 3x^{\frac{2}{3}}y^{\frac{1}{3}} + 3x^{\frac{1}{3}}y^{\frac{2}{3}} &= 144, \\ \text{and } x^{\frac{1}{3}}y^{\frac{1}{3}}\,(x^{\frac{1}{3}} + y^{\frac{1}{3}}) &= 48; \end{aligned}$$

$$\therefore\ x^{\frac{1}{3}}y^{\frac{1}{3}} = \frac{48}{x^{\frac{1}{3}}+y^{\frac{1}{3}}} = 8,$$

$$\text{and } xy = 512.$$

$$\text{Now } x^2 + 2xy + y^2 = 5184;$$

$$\text{and } 4xy = 2048;$$

$$\therefore\ x^2 - 2xy + y^2 = 3136;$$

$$\text{and } \therefore\ x - y = \pm 56,$$

$$\text{but } x + y = 72;$$

$$\therefore\ x = 64 \text{ or } 8.$$

$$\text{and } y = 8 \text{ or } 64.$$

21. Given $\left.\begin{aligned} 4x^2 + \frac{5}{2} &= \frac{x^2}{y} + 10y \\ \text{and } x^2 + 3y &= 55 \end{aligned}\right\}$ to find the values of x and y.

$$\text{by transposition } 4x^2 - \frac{x^2}{y} = 10y - \frac{5}{2};$$

$$\therefore\ \frac{4x^2y - x^2}{y} = \frac{20y - 5}{2};$$

$$\text{or } \frac{x^2}{y}(4y - 1) = \frac{5}{2}(4y - 1);$$

$$\therefore\ \frac{x^2}{y} = \frac{5}{2},$$ unless $y = \frac{1}{4}$

$$\text{and } x^2 = \frac{5y}{2}.$$

Substituting this value in the second equation,

$$\frac{5y}{2} + 3y = 55,$$

$$11y = 110;$$

$$\text{and } y = 10;$$

$$\therefore\ x^2 = \frac{5y}{2} = \frac{50}{2} = 25,$$

$$\text{and } x = \pm 5.$$

22. Given $\frac{1}{x+\sqrt{2-x^2}} + \frac{1}{x-\sqrt{2-x^2}} = ax$, to find the values of x.

$$\text{then } \frac{x-\sqrt{2-x^2}+x+\sqrt{2-x^2}}{2x^2-2} = ax;$$

$$\therefore \frac{2x}{2x^2-2} = ax;$$

$$\frac{1}{x^2-1} = a,$$

$$\text{and } x^2 - 1 = \frac{1}{a},$$

$$x^2 = \frac{1}{a} + 1 = \frac{a+1}{a};$$

$$\therefore x = \pm\sqrt{\frac{a+1}{a}}.$$

23. Given $\frac{x}{\sqrt{a^2+x^2}-x} = b$, to find the values of x.

$$\text{then } x : \sqrt{a^2+x^2} - x :: b : 1,$$

$$x : \sqrt{a^2+x^2} :: b : b+1. \quad (\textit{Alg. } 179.)$$

$$x^2 : a^2 + x^2 :: b^2 : b^2 + 2b + 1;$$

$$x^2 : a^2 :: b^2 : 2b + 1;$$

$$\therefore x^2 = \frac{a^2b^2}{2b+1},$$

$$\text{and } x = \pm\frac{ab}{\sqrt{2b+1}}.$$

24. Given $\left.\begin{array}{l}\sqrt{y} - \sqrt{a-x} = \sqrt{y-x} \\ \text{and } \sqrt{y-x} + \sqrt{a-x} : \sqrt{a-x} :: 5 : 2\end{array}\right\}$ to find the values of x and y.

From the first equation, $\sqrt{y} = \sqrt{y-x} + \sqrt{a-x}$;
Substituting this value in the proportion,

$$\therefore\ \sqrt{y} : \sqrt{a-x} :: 5 : 2;$$

$$\text{but } \sqrt{a-x} : \sqrt{y-x} :: 2 : 3;$$

$$\therefore\ \sqrt{y} : \sqrt{y-x} :: 5 : 3,$$

and $y : y - x :: 25 : 9$. (*Alg.* 188).

$y : x :: 25 : 16$. (*Alg.* 181.);

$$\therefore\ y = \frac{25x}{16},$$

by substitutiou, $\sqrt{\dfrac{25x}{16}} - \sqrt{a-x} = \sqrt{\dfrac{9x}{16}}$,

$$\text{or } \frac{5\sqrt{x} - 3\sqrt{x}}{4} = \sqrt{a-x};$$

$$\therefore\ \frac{\sqrt{x}}{2} = \sqrt{a-x}.$$

Squaring both sides,

$$\frac{x}{4} = a - x,$$

$$\text{and } x = 4a - 4x;$$

$$\therefore\ 5x = 4a,$$

$$\text{and } x = \frac{4a}{5};$$

$$\therefore\ y = \frac{25x}{6} = \frac{100a}{80} = \frac{5a}{4}.$$

25. Given $\left.\begin{array}{l} x^{\frac{4}{3}} + y^{\frac{2}{5}} = 20, \\ \text{and } x^{\frac{2}{3}} + y^{\frac{1}{5}} = 6, \end{array}\right\}$ to find the values of x and y.

Squaring the second equation, we have,

$$x^{\frac{4}{3}} + 2x^{\frac{2}{3}}y^{\frac{1}{5}} + y^{\frac{2}{5}} = 36,$$

$$x^{\frac{4}{3}} \qquad + y^{\frac{2}{5}} = 20.$$

$$\therefore\ 2x^{\frac{2}{3}}y^{\frac{1}{5}} = 16,$$

and $x^{\frac{4}{3}} - 2x^{\frac{2}{3}}y^{\frac{1}{5}} + ^{\frac{2}{5}} = 4$; and, extracting the square root on both sides,

$$x^{\frac{2}{3}} - y^{\frac{1}{5}} = \pm 2,$$

$$\text{but } x^{\frac{2}{3}} + y^{\frac{1}{5}} = 6\,;$$

$$\therefore\ x^{\frac{2}{3}} = 4 \text{ or } 2,$$

$$x = \pm 8 \text{ or } \pm \sqrt{8},$$

$$y^{\frac{1}{5}} = 2 \text{ or } 4,$$

$$\text{and } y = 32 \text{ or } 1024.$$

26. Given $\left.\begin{array}{l} x^4 + 2x^2y^2 + y^4 = 1296 - 4xy\,(x^2 + xy + y^2) \\ \text{and } x - y = 4, \end{array}\right\}$

to find the values of x and y.

$$x^4 + 2x^2y^2 + y^4 = 1296 - 4x^3y + 4x^2y^2 - 4xy^3,$$

$$\therefore\ x^4 + 4x^3y + 6x^2y^2 + 4xy^3 + y^4 = 1296;$$

and extracting the fourth root,

$$x + y = \pm 6,$$

$$\text{but } x - y = \quad 4\,;$$

$$\therefore\ x = 5 \text{ or } -1,$$

$$\text{and } y = 1 \text{ or } -5.$$

27. Given $\dfrac{\sqrt{a} - \sqrt{a-x}}{\sqrt{a} + \sqrt{a-x}} = a$, to find the value of x.

1. Multiplying numerator and denominator by $(\sqrt{a} - \sqrt{a-x})$

$$\frac{(\sqrt{a} - \sqrt{a-x})^2}{x} = a,$$

$$(\sqrt{a} - \sqrt{a-x})^2 = ax\,;$$

$$\text{and } \sqrt{a} - \sqrt{a-x} = \sqrt{ax}\,;$$

$$\therefore\ \sqrt{a} - \sqrt{ax} = \sqrt{a-x}.$$

Squaring both sides,

$$a - 2\sqrt{a^2x} + ax = a - x\,;$$

$$ax + x = 2\sqrt{a^2x}\ ;$$

$\therefore\ \sqrt{x}\,(a + 1) = 2a$ (dividing both sides by $\sqrt{x}$,)

$$x = \frac{4a^2}{(a + 1)^2}.$$

2. Thus, by proportion,

$$\sqrt{a} - \sqrt{a-x} : \sqrt{a} + \sqrt{a-x} :: a : 1,$$

$$\sqrt{a} : \sqrt{a-x} : 1 + a :: 1 - a. \quad (\textit{Alg.}\ 182.)$$

$$a : a - x :: 1 + 2a + a^2 : 1 - 2a + a^2;$$

$$a : x :: 1 + 2a + a^2 : 4a,$$

$$x = \frac{4a^2}{(a+1)^2}.$$

28. Given $\left.\begin{array}{l} \dfrac{\sqrt{x} + \sqrt{x-y}}{\sqrt{x} + \sqrt{x-y}} = 4, \\ \text{and } \sqrt{x} : \sqrt{y} :: \sqrt{y} : 4 \end{array}\right\}$ to find the values of x and y.

$$\sqrt{x} + \sqrt{x-y} : \sqrt{x} - \sqrt{x-y} :: 4 : 1,$$

$$\sqrt{x} : \sqrt{x-y} :: 5 : 3. \quad (\textit{Alg.}\ 182)$$

$$\therefore\ x : x - y :: 25 : 9,$$

$$\text{and } x : y :: 25 : 16;$$

$$\text{but } y : x :: 16 : y;$$

$$\therefore\ y = 25,$$

$$\text{and } x = \frac{y^2}{16} = \frac{625}{16}.$$

29. Given $\left.\begin{array}{l} \dfrac{1}{y} - \dfrac{1}{x} = \dfrac{1}{4} \\ \text{and } x^2y - xy^2 = 16 \end{array}\right\}$ to find the values of x and y.

$x - y = \dfrac{xy}{4}$ (from the first equation,)

and $x - y = \dfrac{16}{xy}$ (from the second equation;)

$$\therefore \frac{xy}{4} = \frac{16}{xy},$$

$$x^2y^2 = 64,$$

$$\text{and } xy = 8.$$

By substitution in the first equation,

$$x - y = 2,$$

$$x^2 - 2xy + y^2 = 4,$$

$$4xy = 32.$$

$$x^2 + 2xy + y^2 = 36$$

$$x + y = \pm 6$$

$$x - y = 2.$$

$$x = 4 \text{ or } -2$$

$$y = 2 \text{ or } -4.$$

30. Given $\dfrac{\sqrt{4x+1} + \sqrt{4x}}{\sqrt{4x+1} - \sqrt{4x}} = 9$, to find the value of x.

$$\sqrt{4x+1} + \sqrt{4x} : \sqrt{4x+1} - \sqrt{4x} :: 9 : 1,$$

$$\sqrt{4x+1} : \sqrt{4x} :: 5 : 4. \quad (\textit{Alg. } 182.)$$

$$4x + 1 : 4x :: 25 : 16;$$

$$4x : 1 :: 16 : 9;$$

$$\therefore 4x = \frac{16}{9},$$

$$\text{and } x = \frac{4}{9}.$$

31. Given $\dfrac{a+x+\sqrt{2ax+x^2}}{a+x-\sqrt{2ax+x^2}} = b$, to find the values of x.

$$a + x + \sqrt{2ax + x^2} : a + x - \sqrt{2ax + x^2} :: b : 1,$$

$$a + x : \sqrt{2ax + x^2} :: b + 1 : b - 1. \quad (\textit{Alg. } 182.)$$

$$a^2 + 2ax + x^2 : 2ax + x^2 :: b^2 + 2b + 1 : b^2 - 2b + 1$$

$$a^2 + 2ax + x^2 : a^2 :: b^2 + 2b + 1 : 4b.$$

$$a+x:a::b+1:\pm 2\sqrt{b},$$

$$x:a::b\mp 2\sqrt{b}+1:\pm 2\sqrt{b};$$

$$\therefore x=\pm a.\frac{(\sqrt{b}\mp 1)^2}{2\sqrt{b}}.$$

32. Given, $\left.\begin{array}{l} x^4y^3 - x^3y^4 = 216 \\ \text{and } x^2y - xy^2 = 6 \end{array}\right\}$ to find the values of x and y.

Dividing the first equation by the second,

$$\text{we have } x^2y^2 = 36;$$

$$\text{and } xy = 6.$$

$$\text{from the second equation } x-y=\frac{6}{xy};$$

$$\therefore x-y=1.$$

Squaring both sides,

$$x^2-2xy+y^2=1,$$
$$4xy=24;$$
$$\therefore x^2+2xy+y^2=25,$$
$$\text{and } x+y=\pm 5;$$
$$\text{but } x-y=1;$$
$$\therefore x=3 \text{ or } -2,$$
$$\text{and } y=2-3.$$

33. Given $\left.\begin{array}{l} x^2+x\sqrt[3]{xy^2} = 208 \\ \text{and } y^2+y\sqrt[3]{x^2y} = 1053 \end{array}\right\}$ to find the values of x and y.

$$x^{\frac{4}{3}}.(x^{\frac{2}{3}}+y^{\frac{2}{3}})=208,$$

$$y^{\frac{4}{3}}.(x^{\frac{2}{3}}+y^{\frac{2}{3}})=1053.$$

Dividing the former by the latter equation,

$$\frac{x^{\frac{4}{3}}}{y^{\frac{4}{3}}}=\frac{208}{1053}=\frac{16}{81},$$

$$\frac{x^{\frac{2}{3}}}{y^{\frac{2}{3}}}=\frac{4}{9}; \text{ and } y^{\frac{2}{3}}=\frac{9x^{\frac{2}{3}}}{4}.$$

Substituting this value of $y^{\frac{2}{3}}$ in the first equation,

$$x^2 + \frac{x^{\frac{4}{3}} \times 9x^{\frac{2}{3}}}{4} = 208;$$

$$\text{or } \frac{13x^2}{4} = 208;$$

$$\therefore\ x^2 = 4 \times 16 = 64,$$

$$\text{and } x = \pm 8;$$

$$\therefore\ y^{\frac{2}{3}} = \frac{9x^{\frac{2}{3}}}{4},$$

$$\text{and } y = \frac{27x}{8} = \pm 27.$$

34. Given $\left.\begin{array}{l} x^{\frac{3}{2}} + x^{\frac{3}{4}}y^{\frac{3}{4}} + y^{\frac{3}{2}} = 1009 \\ \text{and } x^3 + x^{\frac{3}{2}}y^{\frac{3}{2}} + y^3 = 582193 \end{array}\right\}$ to find the values of x and y.

Dividing the second equation by the first,

$$\text{we have } x^{\frac{3}{2}} - x^{\frac{3}{4}}y^{\frac{3}{4}} + y^{\frac{3}{2}} = 577\ (c),$$

$$\text{but } x^{\frac{3}{2}} + x^{\frac{3}{4}}y^{\frac{3}{4}} + y^{\frac{3}{2}} = 1009\ (b);$$

$$\therefore\ 2x^{\frac{3}{4}}y^{\frac{3}{4}} = 432,$$

$$\text{and } x^{\frac{3}{4}}y^{\frac{3}{4}} = 216\ (a),$$

adding (a) to (b) we have

$$x^{\frac{3}{2}} + 2x^{\frac{3}{4}}y^{\frac{3}{4}} + y^{\frac{3}{2}} = 1225$$

$$\text{or } x^{\frac{3}{4}} + y^{\frac{3}{4}} = \pm 35.$$

Subtracting (a) from (c) we have

$$x^{\frac{3}{2}} - 2x^{\frac{3}{4}}y^{\frac{3}{4}} + y^{\frac{3}{2}} = 361,$$

$$\text{or } x^{\frac{3}{4}} - y^{\frac{3}{4}} = \pm 19,$$

$$\text{hence } x^{\frac{3}{4}} + y^{\frac{3}{4}} = \pm 35;$$

$$\text{and } x^{\frac{3}{4}} - y^{\frac{3}{4}} = \pm 19;$$

$$\therefore\ 2x^{\frac{3}{4}} = 54 \text{ or } 16;$$
$$x^{\frac{3}{4}} = 27 \text{ or } 8;$$
$$\text{and } x = 81 \text{ or } 16,$$
$$\text{and } y = 16 \text{ or } 81.$$

35. Given $\left.\begin{array}{r} x^2 + y^2 + xy(x + y) = 68 \\ \text{and } x^3 + y^3 - 3x^2 = 12 + 3y^2 \end{array}\right\}$ to find the values of x and y.

Multiplying the first equation by 3,

$$3x^2 + 3y^2 + 3x^2y + 3xy^2 = 204,$$
$$\text{and } x^3 + y^3 - 3x^2 - 3y^2 = 12.$$
$$\text{then } x^3 + 3x^2y + 3xy^2 + y^3 = 216,$$
$$x + y = 6.$$

Substituting this value for $\overline{x + y}$ in the first equation;

$$\therefore\ x^2 + 6xy + y^2 = 68,$$
$$\text{but } x^2 + 2xy + y^2 = 36.$$
$$4xy = 32,$$
$$\text{then } x^2 - 2xy + y^2 = 4,$$
$$\text{and } x - y = \pm 2,$$
$$\text{but } x + y = 6;$$
$$\therefore\ x = 4 \text{ or } 2,$$
$$\text{and } y = 2 \text{ or } 4.$$

36. Given $\left.\begin{array}{r} xy.\overline{x + y} = 84, \\ \text{and } x^2y^2.\overline{x^2+y^2} = 3600, \end{array}\right\}$ to find the values of x and y.

From the first equation $x^2y + xy^2 = 84$;

$$\therefore\ x^4y^2 + 2x^3y^3 + x^2y^4 = 7056.$$

From the second equation $x^4y^2 + x^2y^4 = 3600$;

$$\therefore\ 2x^3y^3 = 3456,$$
$$x^3y^3 = 1728,$$
$$\text{and } xy = 12.$$

$$\text{Now } x + y = \frac{84}{xy} = 7;$$

$$\therefore x^2 + 2xy + y^2 = 49,$$
$$4xy = 48;$$

$$\therefore x^2 - 2xy + y^2 = 1,$$
$$\text{and } x - y = \pm 1,$$
$$\text{but } x + y = 7;$$

$$\therefore x = 4 \text{ or } 3,$$
$$\text{and } y = 3 \text{ or } 4.$$

87. Given $\dfrac{x^2 + xy + y^2}{x + y} = 7\ (a),$ and $\dfrac{x^2 - xy + y^2}{x - y} = 9\ (b),$ to find the values of x and y.

Dividing (a) by (b) we have,

$$\frac{x^3 - y^3}{x^3 + y^3} = \frac{7}{9};$$

$$\therefore x^3 + y^3 : x^3 - y^3 :: 9 : 7$$
$$x^3 : y^3 :: 16 : 2 \quad (\textit{Alg. } 182.)$$
$$x^3 : y^3 :: 8 : 1,$$
$$x : y :: 2 : 1,$$
$$\text{and } x = 2y;$$

$$\therefore \frac{4y^2 + 2y^2 + y^2}{3y} = 7,$$

$$\text{and } \frac{7y}{3} = 7;$$

$$\therefore y = 3,$$
$$\text{and } x = 2y = 6.$$

Solutions of Adfected Quadratics, involving only one unknown Quantity.

36. Given $\dfrac{\sqrt{x}+9}{\sqrt{x}} = \dfrac{\sqrt{9x}-3\frac{4}{5}}{9-\sqrt{x}}$, to find the values of x.

$$\text{then } 81 - x = 3x - \frac{19\sqrt{x}}{5},$$

$$\text{or } 4x - \frac{19\sqrt{x}}{5} = 81;$$

$$\therefore\ x - \frac{19\sqrt{x}}{20} + \frac{361}{1600} = \frac{81}{4} + \frac{361}{1600} = \frac{32400+361}{1600} = \frac{32761}{1600};$$

$$\therefore\ \sqrt{x} - \frac{19}{40} = \pm\frac{181}{40},$$

$$\sqrt{x} = \frac{200}{40} \text{ or } -\frac{162}{40},$$

$$\sqrt{x} = 5 \text{ or } -\frac{81}{20};$$

$$\therefore\ x = 25 \text{ or } \frac{6561}{400}.$$

37. Given $\dfrac{x+\sqrt{x}}{x-\sqrt{x}} = \dfrac{x^2-x}{4}$, to find the values of x.

Dividing both sides by $x+\sqrt{x}$,

$$\frac{1}{x-\sqrt{x}} = \frac{x-\sqrt{x}}{4};$$

$$\therefore\ (x-\sqrt{x})^2 = 4,$$

$$\text{and } x - \sqrt{x} = \pm 2.$$

Completing the square,

$$x - \sqrt{x} + \frac{1}{4} = \frac{1}{4} \pm 2 = \frac{9}{4} \text{ or } \frac{-7}{4};$$

$$\therefore\ \sqrt{x} - \frac{1}{2} = \pm\frac{3}{2} \text{ or } \pm\frac{\sqrt{-7}}{2},$$

$$\text{and } \sqrt{x} = 2 \text{ or } -1 \text{ or } \frac{1 \pm \sqrt{-7}}{2}.$$

Squaring both sides,

$$x = 4 \text{ or } 1 \text{ or } \frac{1 \pm 2\sqrt{-7} - 7}{4},$$

$$= 4 \text{ or } 1 \text{ or } \frac{-6 \pm 2\sqrt{-7}}{4},$$

$$= 4 \text{ or } 1 \text{ or } \frac{-3 \pm \sqrt{-7}}{2}.$$

38. Given $\dfrac{x - \sqrt{x+1}}{x + \sqrt{x+1}} = \dfrac{5}{11}$, to find the values of x.

$$x + \sqrt{x+1} : x - \sqrt{x+1} :: 11 : 5,$$

$$\text{then } x : \sqrt{x+1} :: 8 : 3;$$

$$\therefore\ x^2 : x+1 :: 64 : 9,$$

$$\text{and } x^2 = \frac{64}{9}x + \frac{64}{9};$$

$$\therefore\ x^2 - \frac{64x}{9} + \overline{\frac{32}{9}}\Big|^2 = \frac{1024}{81} + \frac{64}{9} = \frac{1600}{81},$$

$$\text{then } x - \frac{32}{9} = \pm\frac{40}{9};$$

$$\text{and } \therefore\ x = 8 \text{ or } -\frac{8}{9}$$

39. Given $5.\dfrac{3x-1}{1+5\sqrt{x}} + \dfrac{2}{\sqrt{x}} = 3\sqrt{x}$, to find the values of x.

$$\text{Now } 15x - 5 + \frac{2 + 10\sqrt{x}}{\sqrt{x}} = 3\sqrt{x} + 15x;$$

$$\therefore\ 3x + 5\sqrt{x} = 10\sqrt{x} + 2,$$

$$\text{and } 3x - 5\sqrt{x} = 2,$$

then $x - \dfrac{5\sqrt{x}}{3} = \dfrac{2}{3}$, and completing the square,

$$x - \frac{5\sqrt{x}}{3} + \frac{25}{36} = \frac{24 + 25}{36} = \frac{49}{36};$$

$$\therefore \sqrt{x} - \frac{5}{6} = \pm\frac{7}{6},$$

$$\sqrt{x} = 2 \text{ or } -\frac{1}{3},$$

$$\text{and } x = 4 \text{ or } \frac{1}{9}.$$

40. Given $\sqrt{x^5} - \dfrac{40}{\sqrt{x}} = 3x$, to find the values of x.

Clearing of fractions,

$$x^3 - 40 = 3x^{\frac{3}{2}},$$

then $x^3 - 3x^{\frac{3}{2}} = 40$; and completing the square,

$$x^3 - 3x^{\frac{3}{2}} + \overline{\tfrac{3}{2}}\Big|^2 = \frac{9}{4} + 40 = \frac{169}{4};$$

$$\therefore x^{\frac{3}{2}} - \frac{3}{2} = \pm\frac{13}{2},$$

$$x^{\frac{3}{2}} = 8 \text{ or } -5,$$

$$\text{and } x = 4 \text{ or } \overline{-5}|^{\frac{2}{3}}.$$

41. Given $x^{\frac{4}{3}} + 7x^{\frac{2}{3}} = 44$, to find the values of x.

Here $x^{\frac{4}{3}} + 7x^{\frac{2}{3}} + \dfrac{49}{4} = 44 + \dfrac{49}{4} = \dfrac{225}{4}$

$$\text{then } x^{\frac{2}{3}} + \frac{7}{2} = \pm\frac{15}{2},$$

$$\text{and } x^{\frac{2}{3}} = 4 \text{ or } -11;$$

$$\therefore x = \pm 8 \text{ or } \overline{-11}|^{\frac{3}{2}}.$$

42. Given $4x^{\frac{1}{3}} + x^{\frac{1}{6}} = 39$, to find the values of x.

Dividing by 4, $x^{\frac{1}{3}} + \dfrac{x^{\frac{1}{6}}}{4} = \dfrac{39}{4}$,

$$\text{then } x^{\frac{1}{3}} + \frac{x^{\frac{1}{6}}}{4} + \frac{1}{64} = \frac{625}{64},$$

$$\text{and } x^{\frac{1}{6}} + \frac{1}{8} = \pm\frac{25}{8};$$

$$\therefore\ x^{\frac{1}{6}} = 3 \text{ or } -\frac{13}{4},$$

$$\text{and } x = 729 \text{ or } \left(\frac{-13}{4}\right)^6.$$

43. Given $3x^6 + 42x^3 = 3321$, to find the values of x.

$$\text{Now } x^6 + 14x^3 = 1107;$$

$$\therefore\ x^6 + 14x^3 + 49 = 1107 + 49 = 1156,$$

$$\text{and } x^3 + 7 = \pm 34;$$

$$\therefore\ x^3 = 27 \text{ or } -41,$$

$$\text{and } x = 3 \text{ or } -\sqrt[3]{41}.$$

44. Given $\dfrac{8}{x^3} + 2 = \dfrac{17}{x^{\frac{3}{2}}}$, to find the values of x.

Dividing by 8, and transposing

$$\frac{1}{x^3} - \frac{17}{8x^{\frac{3}{2}}} = -\frac{1}{4},$$

$$\frac{1}{x^3} - \frac{17}{8x^{\frac{3}{2}}} + \frac{289}{256} = \frac{289}{256} - \frac{1}{4} = \frac{225}{256},$$

$$\frac{1}{x^{\frac{3}{2}}} - \frac{17}{16} = \pm\frac{15}{16},$$

$$\frac{1}{x^{\frac{3}{2}}} = 2 \text{ or } \frac{1}{8};$$

$$\therefore\ x^{\frac{3}{2}} = 8 \text{ or } \frac{1}{2},$$

$$x = 4 \text{ or } \left(\frac{1}{4}\right)^{\frac{1}{3}}$$

45. Given $x^{\frac{7}{3}} + \frac{41\sqrt[3]{x}}{x} = \frac{97}{\sqrt[3]{x^2}} + x^{\frac{5}{6}}$, to find the values of x.

$$\text{or } x^{\frac{7}{3}} + \frac{41}{x^{\frac{2}{3}}} = \frac{97}{x^{\frac{2}{3}}} + x^{\frac{5}{6}},$$

$$\text{or } x^3 + 41 = 97 + x^{\frac{3}{2}},$$

$$\therefore x^3 - x^{\frac{3}{2}} = 56,$$

$$x^3 - x^{\frac{3}{2}} + \frac{1}{4} = \frac{224 + 1}{4} = \frac{225}{4};$$

$$\therefore x^{\frac{3}{2}} - \frac{1}{2} = \pm \frac{15}{2},$$

$$\text{and } x^{\frac{3}{2}} = 8 \text{ or } -7;$$

$$\therefore x = 4 \text{ or } \overline{-7)}^{\frac{2}{3}}.$$

46. Given $\sqrt[6]{\frac{1}{x^4}} + \sqrt[3]{\frac{1}{x}} = \frac{3 - \sqrt[3]{x^2}}{x}$, to find the values of x.

$$\frac{1}{x^{\frac{2}{3}}} + \frac{1}{x^{\frac{1}{3}}} = \frac{3 - x^{\frac{2}{3}}}{x},$$

$$\text{then } x^{\frac{1}{3}} + x^{\frac{2}{3}} = 3 - x^{\frac{2}{3}},$$

$$2x^{\frac{2}{3}} + x^{\frac{1}{3}} = 3,$$

$$x^{\frac{2}{3}} + \frac{x^{\frac{1}{3}}}{2} + \frac{1}{16} = \frac{24 + 1}{16} = \frac{25}{16},$$

$$x^{\frac{1}{3}} + \frac{1}{4} = \pm \frac{5}{4},$$

$$x^{\frac{1}{3}} = 1 \text{ or } -\frac{3}{2};$$

$$\therefore x = 1 \text{ or } -\frac{27}{8}.$$

47. Given $3x^n \sqrt[3]{x^n} - \dfrac{4x^n}{\sqrt[3]{x^n}} = 4$, to find the values of x.

$$3x^{\frac{4n}{3}} - 4x^{\frac{2n}{3}} = 4,$$

$$x^{\frac{4n}{3}} - \frac{4x^{\frac{2n}{3}}}{3} = \frac{4}{3},$$

$$x^{\frac{4n}{3}} - \frac{4x^{\frac{2n}{3}}}{3} + \frac{4}{9} = \frac{12+4}{9} = \frac{16}{9};$$

$$x^{\frac{2n}{3}} - \frac{2}{3} = \pm\frac{4}{3},$$

$$x^{\frac{2n}{3}} = 2 \text{ or } -\frac{2}{3},$$

$$x^{2n} = 8 \text{ or } -\frac{8}{27};$$

$$\text{and } x = (8)^{\frac{1}{2n}} \text{ or } \left(-\frac{8}{27}\right)^{\frac{1}{2n}}.$$

48. Given $adx - acx^2 = bcx - bd$, to find the values of x.

$$acx^2 + bcx - adx = bd$$

$$x^2 + \frac{bc-ad}{ac}x = \frac{bd}{ac}$$

$$x^2 + \frac{bc-ad}{ac}x + \left(\frac{bc-ad}{2ac}\right)^2 = \frac{bd}{ac} + \frac{b^2c^2 + 2abcd + a^2d^2}{4a^2c^2}$$

$$= \frac{4abcd + b^2c^2 - 2abcd + a^2d^2}{4a^2c^2}$$

$$= \frac{b^2c^2 + 2abcd + a^2d^2}{4a^2c^2};$$

$$\therefore x + \frac{bc-ad}{2ac} = \pm\frac{bc+ad}{2ac};$$

$$\text{and } x = \frac{2ad}{2ac} \text{ or } -\frac{2bc}{2ac}$$

$$= \frac{d}{c} \text{ or } -\frac{b}{a}.$$

49. Given $\frac{a^2x^2}{b^2} - \frac{2ax}{c} + \frac{d^2}{c^2} = 0$, to find the values of x.

$$a^2x^2 - \frac{2ab^2x}{c} = - \frac{b^2d^2}{c^2},$$

$$a^2x^2 - \frac{2ab^2x}{c} + \frac{b^4}{c^2} = \frac{b^4}{c^2} - \frac{b^2d^2}{c^2};$$

$$\therefore ax - \frac{b^2}{c} = \frac{\pm\sqrt{b^4 - b^2d^2}}{c} = \frac{\pm b\sqrt{b^2 - d^2}}{c}$$

$$ax = \frac{b^2 \pm b\sqrt{b^2 - d^2}}{c} = b.\frac{b \pm \sqrt{b^2 - d^2}}{c}$$

$$\therefore x = \frac{b}{a}.\ \frac{b \pm \sqrt{b^2 - d^2}}{c}.$$

50. Given, $9a^4b^4x^2 - 6a^3b^2x = b^2$ to find the values of x.

$$x^2 - \frac{2x}{3ab^2} = \frac{1}{9a^4b^2},$$

$$x^2 - \frac{2x}{3ab^2} + \frac{1}{9a^2b^4} = \frac{1}{9a^2b^4} + \frac{1}{9a^4b^2}$$

$$= \frac{a^2 + b^2}{9a^4b^4};$$

$$x - \frac{1}{3ab^2} = \frac{\pm\sqrt{a^2 + b^2}}{3a^2b^2},$$

$$x = \frac{a \pm \sqrt{a^2 + b^2}}{3a^2b^2}.$$

51. Given, $\overline{a+b}.x^2 = cx + \frac{ac}{a+b}$, to find the values of x.

$$\overline{a+b}.x^2 - cx = \frac{ac}{a+b};$$

$$x^2 - \frac{cx}{a+b} = \frac{ac}{\overline{a+b}|^2};$$

$$x^2 - \frac{cx}{a+b} + \frac{c^2}{4.\overline{a+b}|^2} = \frac{ac}{\overline{a+b}|^2} + \frac{c^2}{4.\overline{a+b}|^2} = \frac{c^2+4ac}{4.\overline{a+b}|^2}$$

$$x - \frac{c}{2(a+b)} = \pm\frac{\sqrt{c^2+4ac}}{2(a+b)};$$

$$\text{and } x = \frac{c \pm \sqrt{c^2+4ac}}{2(a+b)}.$$

The succeeding five equations merely require attention to the ordinary rules, as laid down by Dr. Bland; they are long and tedious, without a single point worthy of notice, and would consequently occupy room to no purpose.

57. Given $x + \sqrt{x} : x - \sqrt{x} :: 3\sqrt{x}+6 : 2\sqrt{x}$ to find the values of x.

$$\text{then } \sqrt{x}+1 : \sqrt{x}-1 :: 3\sqrt{x}+6 : 2\sqrt{x};$$

$$\text{or } 2x + 2\sqrt{x} = 3x + 3\sqrt{x} - 6,$$

$$x + \sqrt{x} = 6;$$

$$x + \sqrt{x} + \frac{1}{4} = \frac{25}{4}$$

$$\sqrt{x} + \frac{1}{2} = \pm\frac{5}{2},$$

$$\sqrt{x} = 2 \text{ or } 3;$$

$$\therefore x = 4 \text{ or } 9.$$

58. Given $x^2 + 11 + \sqrt{x^2+11} = 42$, to find the values of x.

$$x^2 + 11 + \sqrt{x^2+11} + \frac{1}{4} = \frac{169}{4},$$

$$\sqrt{x^2+11} + \frac{1}{2} = \pm\frac{13}{2}$$

$$\sqrt{x^2+11} = 6 \text{ or } -7,$$

$$x^2 + 11 = 36 \text{ or } 49,$$
$$x^2 = 25 \text{ or } 38;$$
$$\therefore\ x = \pm 5 \text{ or } \pm \sqrt{38}$$

59. Given, $\overline{x-5}|^3 - 3\,.\,\overline{x-5}|^{\frac{3}{2}} = 40$, to find the values of x.

$$\overline{x-5}|^3 - 3\,.\,\overline{x-5}|^{\frac{3}{2}} + \frac{9}{4} = 40 + \frac{9}{4} = \frac{169}{4}$$
$$\overline{x-5}|^{\frac{3}{2}} - \frac{3}{2} = \pm \frac{13}{2}$$
$$\overline{x-5}|^{\frac{3}{2}} = 8 \text{ or } -5$$
$$x - 5 = 4 \text{ or } \overline{-5}|^{\frac{2}{3}}$$
$$x = 9 \text{ or } \overline{-5}|^{\frac{2}{3}} + 5.$$

60. Given $x + \sqrt{x+6} = 2 + 3\sqrt{x+6}$, to find the values of x.

$$x - 2\sqrt{x+6} = 2$$
$$x + 6 - 2\sqrt{x+6} = 8$$
$$x + 6 - 2\sqrt{x+6} + 1 = 9$$
$$\sqrt{x+6} - 1 = \pm 3$$
$$\sqrt{x+6} = 4 \text{ or } -2$$
$$x + 6 = 16 \text{ or } 4$$
$$\text{and } x = 10 \text{ or } -2.$$

61. Given $\overline{x^2+5}|^2 - 4x^2 = 160$, to find the values of x.

$$\text{then } \overline{x^2+5}|^2 - 4x^2 - 20 = 140$$
$$\overline{x^2+5}|^2 - 4(x^2+5) + 4 = 144$$
$$x^2 + 5 - 2 = \pm 12$$
$$x^2 + 5 = 14 \text{ or } -10$$
$$x^2 = 9 \text{ or } -15$$
$$\text{and } x = \pm 3 \pm \sqrt{-15}.$$

62. Given $x^2 - 7x + \sqrt{x^2-7x+18} = 24$, to find the values of x.

$$x^2 - 7x + 18 + \sqrt{x^2-7x+18} + \frac{1}{4} = 42 + \frac{1}{4} = \frac{169}{4}$$

$$\therefore \sqrt{x^2-7x+18} + \frac{1}{2} = \pm\frac{13}{2}$$

$$\sqrt{x^2-7x+18} = 6 \text{ or } -7,$$

$$\text{and } x^2 - 7x + 18 = 36 \text{ or } 49.$$

$$\therefore x^2 - 7x = 18 \text{ or } 31,$$

$$x^2 - 7x + \frac{49}{4} = \frac{121}{4} \text{ or } \frac{173}{4}$$

$$x - \frac{7}{2} = \pm\frac{11}{2} \text{ or } \pm\frac{\sqrt{173}}{2};$$

$$\therefore x = 9 \text{ or } 2 \text{ or } \frac{7 \pm \sqrt{173}}{2}.$$

63. Given $9x - 4x^2 + \sqrt{4x^2-9x+11} = 5$, to find the values of x.

$$\text{Now } 4x^2 - 9x - \sqrt{4x^2-9x+11} = -5,$$

$$4x^2 - 9x + 11 - \sqrt{4x^2-9x+11} + \frac{1}{4} = 6 + \frac{1}{4} = \frac{25}{4}$$

$$\therefore \sqrt{4x^2-9x+11} - \frac{1}{2} = \pm\frac{5}{2}$$

$$\sqrt{4x^2-9x+11} = 3 \text{ or } -2$$

$$\text{and } 4x^2 - 9x + 11 = 9 \text{ or } 4,$$

$$\therefore 4x^2 - 9x = -2 \text{ or } -7$$

$$x^2 - \frac{9x}{4} = -\frac{2}{4} \text{ or } -\frac{7}{4}$$

$$x^2 - \frac{9x}{4} + \frac{81}{64} = \frac{49}{64} \text{ or } -\frac{31}{64}$$

$$x - \frac{9}{8} = \pm\frac{7}{8} \text{ or } \frac{\pm\sqrt{-31}}{8}$$

$$\therefore\ x = 2 \text{ or } \frac{1}{4} \text{ or } \frac{9 \pm \sqrt{-31}}{8}.$$

64. Given $x^2 + \sqrt{5x + x^2} = 42 - 5x$, to find the values of x.

$$\text{Here } 5x + x^2 + \sqrt{5x + x^2} = 42$$

$$5x + x^2 + \sqrt{5x + x^2} + \frac{1}{4} = \frac{169}{4}$$

$$\sqrt{5x + x^2} + \frac{1}{2} = \pm\frac{13}{2}$$

$$\sqrt{5x + x^2} = 6 \text{ or } - 7,$$

$$\therefore\ 5x + x^2 = 36 \text{ or } 49$$

$$\text{Again, } x^2 + 5x + \frac{25}{4} = 36 + \frac{25}{4} \text{ or } 49 + \frac{25}{4}$$

$$= \frac{169}{4} \text{ or } \frac{221}{4}$$

$$x + \frac{5}{2} = \pm\frac{13}{2} \text{ or } \frac{\pm\sqrt{221}}{2}$$

$$= 4 \text{ or } - 9 \text{ or } \frac{-5 \pm \sqrt{221}}{2}$$

65. Given $\dfrac{2}{\overline{x+2}|^{\frac{3}{2}}} + \dfrac{\sqrt{x+2}}{2} = \dfrac{17}{4\sqrt{x+2}}$, to find the values of x.

$$\frac{1}{\overline{x+2}|^2} + \frac{1}{4} = \frac{17}{8(x+2)}$$

$$\frac{1}{(x + 2)^2} - \frac{17}{8(x + 2)} = -\frac{1}{4},$$

$$\frac{1}{(x + 2)^2} - \frac{17}{8(x + 2)} + \frac{289}{256} = \frac{289}{256} - \frac{1}{4} = \frac{225}{256}$$

$$\frac{1}{x+2} - \frac{17}{16} = \pm\frac{15}{16}$$

$$\frac{1}{x+2} = 2 \text{ or } \frac{1}{8}$$

$$x + 2 = 8 \text{ or } \frac{1}{2}$$

$$x = 6 \text{ or } -\frac{3}{2}.$$

66. Given $\frac{x}{x+4} + \frac{4}{\sqrt{x+4}} = \frac{21}{x}$, to find the values of x.

Then $\frac{x^2}{x+4} + \frac{4x}{\sqrt{x+4}} = 21$

$$\frac{x^2}{x+4} + \frac{4x}{\sqrt{x+4}} + 4 = 25$$

$$\frac{x}{\sqrt{x+4}} + 2 = \pm 5$$

$$\frac{x}{\sqrt{x+4}} = 3 \text{ or } -7$$

$$\frac{x^2}{x+4} = 9 \text{ or } 49.$$

$\therefore x^2 = 9x + 36$ or $x^2 = 49x + 196$

$x^2 - 9x + \frac{81}{4} = \frac{225}{4}$ $\qquad$ $x^2 - 49x + \frac{2401}{4} = \frac{3185}{4}$

$x - \frac{9}{2} = \pm \frac{15}{2}$ $\qquad$ $x - \frac{49}{2} = \frac{\pm\sqrt{3185}}{2}$

$$x = 12 \text{ or } -3, \text{ or } \frac{49 \pm \sqrt{3185}}{2}.$$

67. Given $\frac{3x+5}{3x-5} - \frac{3x-5}{3x+5} = \frac{135}{176}$, to find the values of x.

then $\frac{9x^2 + 30x + 25 - 9x^2 + 30x - 25}{9x^2 - 25} = \frac{135}{176}$,

$$\text{or } \frac{60x}{9x^2 - 25} = \frac{135}{176};$$

$$\therefore \frac{4x}{9x^2-25} = \frac{9}{176},$$

$$81x^2 - 225 = 704x,$$

$$81x^2 - 704x = 225,$$

$$x^2 - \frac{704}{81}x = \frac{225}{81},$$

$$x^2 - \frac{704}{81}x + \frac{123904}{6561} = \frac{123904}{6561} + \frac{225}{81} = \frac{123904 + 18225}{6561}$$

$$= \frac{142129}{6561}$$

$$\therefore x - \frac{352}{81} = \pm \frac{377}{81}$$

and $x = 9$ or $-\frac{25}{81}$.

68. Given $x + \sqrt{x} + 2 = \frac{x^2+x-4}{\sqrt{x}}$, to find the values of x.

Then $x + \sqrt{x} + 2 = \frac{x^2 - 4}{\sqrt{x}} + \sqrt{x}$;

$$\therefore x + 2 = \frac{x^2 - 4}{\sqrt{x}},$$

and $\sqrt{x} = x - 2$,

$$x - \sqrt{x} = 2,$$

$$x - \sqrt{x} + \frac{1}{4} = \frac{9}{4}$$

$$\sqrt{x} - \frac{1}{2} = \pm\frac{3}{2},$$

$$\sqrt{x} = 2 \text{ or } - 1$$

$$\therefore x = 4 \text{ or } 1.$$

69. Given $\frac{x^2}{\overline{x^2-4}|^2} + \frac{6}{x^2-4} = \frac{351}{25x^2}$, to find the values of x.

Then $\frac{25x^4}{\overline{x^2-4}|^2} + \frac{30 \times 5x^2}{x^2-4} + 225 = 351 + 225 = 576,$

$$\frac{5x^2}{x^2-4} + 15 = \pm 24,$$

$$\frac{5x^2}{x^2-4} = 9 \text{ or } -39;$$

$\therefore\ 5x^2 = 9x^2 - 36$

$4x^2 = 36$

$x^2 = 9$

$x = \pm 3$

or $5x^2 = -39x^2 + 156$

$44x^2 = 156$

$x^2 = \frac{156}{44} = \frac{39}{11}$

$x = \pm\sqrt{\frac{39}{11}}.$

70. Given $\overline{x+\frac{8}{x}}|^2 + x = 42 - \frac{8}{x}$, to find the values of x.

Here $\overline{x+\frac{8}{x}}|^2 + \left(x+\frac{8}{x}\right) + \frac{1}{4} = 42 + \frac{1}{4} = \frac{169}{4}.$

$$x + \frac{8}{x} + \frac{1}{2} = \pm\frac{13}{2}$$

$$x + \frac{8}{x} = 6 \text{ or } -7:$$

$\therefore\ x^2 + 8 = 6x$

$x^2 - 6x = -8$

$x^2 - 6x + 9 = 1$

$x - 3 = \pm 1$

$\therefore\ x = 4 \text{ or } 2$

or $x^2 + 8 = -7x$

$x^2 + 7x = -8$

$x^2 + 7x + \frac{49}{4} = \frac{49}{4} - \frac{32}{4} = \frac{17}{4}$

$x + \frac{7}{2} = \frac{\pm\sqrt{17}}{2}$

$\therefore\ x = \frac{-7 \pm \sqrt{17}}{2}.$

71. Given $x + 4 - 2\sqrt{\dfrac{x+4}{x-4}} = \dfrac{3}{x-4}$, to find the values of x.

$$(x^2 - 16) - 2\sqrt{x^2 - 16} = 3,$$

$$(x^2 - 16) - 2\sqrt{x^2 - 16} + 1 = 4,$$

$$\sqrt{x^2 - 16} - 1 = \pm 2.$$

$$\sqrt{x^2 - 16} = 3 \text{ or } -1,$$

$$x^2 - 16 = 9 \text{ or } 1,$$

$$x^2 = 25 \text{ or } 17;$$

$$\therefore x = \pm 5 \text{ or } \pm \sqrt{17}$$

72. Given $x^4\left(1 + \dfrac{1}{3x}\right)^2 - (3x^3 + x) = 70$, to find the values of x.

$$x^4\left(1 + \frac{1}{3x}\right)^2 - 3x^2\left(1 + \frac{1}{3x}\right) + \frac{9}{4} = 70 + \frac{9}{4} = \frac{289}{4}$$

$$x^2\left(1 + \frac{1}{3x}\right) - \frac{3}{2} = \pm \frac{17}{2},$$

$$x^2\left(1 + \frac{1}{3x}\right) = 10 \text{ or } -7;$$

$$\therefore x^2 + \frac{x}{3} = 10 \text{ or } -7,$$

$$x^2 + \frac{x}{3} + \frac{1}{36} = \frac{361}{36} \text{ or } -\frac{251}{36},$$

$$x + \frac{1}{6} = \pm \frac{19}{6} \text{ or } \pm \frac{\sqrt{251}}{6},$$

$$x = 3 \text{ or } -\frac{10}{3} \text{ or } \frac{-1 \pm \sqrt{-251}}{6}.$$

73. Given $x^2 - \dfrac{5x}{2} + 15 = \dfrac{25x^2}{16} - \dfrac{64}{x^2}$, to find the values of x.

$$x^2 15 + \frac{64}{x^2} = \frac{25x^2}{16} + \frac{5x}{2}$$

$$\text{then } x^2 + 16 + \frac{64}{x^2} = \frac{25x^2}{16} + \frac{5x}{2} + 1,$$

$$x + \frac{8}{x} = \pm\left(\frac{5x}{4} + 1\right),$$

$$\text{or } 4x^2 + 32 = -5x^2 - 4x \quad \frac{x}{4} + 1 = \frac{8}{x}$$

$$x^2 + 4x = 32 \qquad 9x^2 + 4x = -32$$

$$x^2 + 4x + 4 = 36 \qquad x^2 + \frac{4x}{9} + \frac{4}{81} = \frac{4}{81} - 32 = \frac{-284}{81}$$

$$x + 2 = \pm 6 \qquad x + \frac{2}{9} = \pm\frac{\sqrt{-284}}{9} \text{ or } \frac{\pm 2\sqrt{-71}}{9}$$

$$x = 4 \text{ or } -8 \text{ or } \frac{-2 \pm 2\sqrt{-71}}{9}$$

74. Given $\frac{35\frac{5}{7}}{\sqrt{x^4 - 9x^2}} + \frac{\sqrt{x^2 - 9}}{7x} = \frac{19}{2x}$, to find the values of x.

$$\text{or } \frac{250}{7\sqrt{x^2 - 9}} + \frac{\sqrt{x^2 - 9}}{7} = \frac{19}{2},$$

$$250 + (x^2 - 9) = \frac{133\sqrt{x^2 - 9}}{2},$$

$$(x^2 - 9) - \frac{133.\sqrt{x^2 - 9}}{2} + \left|\frac{133}{4}\right|^2 = \frac{17689}{16} - 250 = \frac{17689 - 4000}{16}$$

$$= \frac{13689}{16}$$

$$\sqrt{x^2 - 9} - \frac{133}{4} = \frac{\pm 117}{4},$$

$$\sqrt{x^2 - 9} = \frac{16}{4} \text{ or } \frac{250}{4}$$

$$= 4 \text{ or } \frac{125}{2}$$

$$x^2 - 9 = 16 \text{ or } \frac{15625}{4}$$

$$x^2 = 25 \text{ or } \frac{15661}{4}$$

$$x = \pm 5 \text{ or } \pm \frac{\sqrt{15661}}{2}.$$

75. Given $3.\overline{\overline{x-1}|^2 - x}|^2 + 2x = 341 + 2.\overline{x-1}|^2$, to find the values of x.

Then $3.\overline{\overline{x-1}|^2 - x}|^2 - 2.\overline{\overline{x-1}|^2 - x} = 341,$

$$\overline{\overline{x-1}|^2 - x}|^2 - \frac{2}{3}.\overline{\overline{x-1}|^2 - x} + \frac{1}{9} = \frac{341}{3} + \frac{1}{9} = \frac{1024}{9}$$

$$(\overline{x-1}|^2 - x) - \frac{1}{3} = \pm \frac{32}{3}$$

$$\overline{x-1}|^2 - x = 11 \text{ or } -\frac{31}{3}$$

$$x^2 - 2x + 1 - x = 11 \text{ or } -\frac{31}{3}$$

$$x^2 - 3x = 10 \text{ or } -\frac{34}{3}$$

$$x^2 - 3x + \frac{9}{4} = \frac{49}{4} \text{ or } \frac{-109}{12}$$

$$x - \frac{3}{2} = \pm \frac{7}{2} \text{ or } \frac{\pm\sqrt{-109}}{2\sqrt{3}}$$

$$\therefore x = 5 \text{ or } -2 \text{ or } \frac{3}{2} \pm \frac{\sqrt{-109}}{2\sqrt{3}} = \frac{3\sqrt{3} \pm \sqrt{-109}}{2\sqrt{3}}.$$

76. Given $x^4 + \frac{13}{3}x^3 - 39x = 81$, to find the values of x.

Here $x^4 + \frac{13}{3}x^3 = 39x + 81,$

$$\therefore x^4 + \frac{13x^3}{3} + \frac{169x^2}{36} = \frac{169x^2}{36} + 39x + 81,$$

$$x^2 + \frac{13x}{6} = \pm\left(\frac{13x}{6} + 9\right),$$

and first, $x^2 = 9$,

$$\therefore\ x = \pm 3,$$

secondly, $x^2 + \dfrac{26x}{6} = -9$,

$$x^2 + \frac{26x}{6} + \frac{169}{36} = \frac{169}{36} - 9 = \frac{169-324}{36} = \frac{-155}{36}$$

$$x + \frac{13}{6} = \frac{\pm\sqrt{-155}}{6}$$

$$x = \frac{-13 \pm \sqrt{-155}}{6}.$$

77. Given $4x + \dfrac{x}{2} = 4x^3 + 33$, to find the values of x.

Here $4x^4 - 4x^3 = 33 - \dfrac{x}{2}$

$$4x^4 - 4x^3 + x^2 = 33 + x^2 - \frac{x}{2} = 33 + \frac{2x^2 - x}{2}$$

$$\overline{2x^2 - x}|^2 - \frac{2x^2 - x}{2} + \frac{1}{16} = 33 + \frac{1}{16} = \frac{529}{16}$$

$$2x^2 - x - \frac{1}{4} = \pm\frac{23}{4}$$

$$2x^2 - x = 6 \text{ or } -\frac{11}{2}$$

$$x^2 - \frac{x}{2} = 3 \text{ or } -\frac{11}{4}$$

$$x^2 - \frac{x}{2} + \frac{1}{16} = \frac{49}{16} \text{ or } -\frac{43}{16}$$

$$x - \frac{1}{4} = \pm\frac{7}{4} \text{ or } \frac{\pm\sqrt{-43}}{4}$$

$$\text{and } x = 2 \text{ or } -\frac{3}{2} \text{ or } \frac{1 \pm \sqrt{-43}}{4}.$$

78. Given $\overline{x-2}|^2 - 6x^{\frac{1}{2}}(x-2) = 24 - 14x + 15x^{\frac{1}{2}}$ to find the values of x.

$$\overline{x-2}|^2 - 6x^{\frac{1}{2}}(x-2) + 9x = 24 - 5x + 15x^{\frac{1}{2}},$$

$$\therefore\ x - 2 - 3x^{\frac{1}{2}} = \pm\sqrt{24 - 5(x - 3x^{\frac{1}{2}})}$$

$$\text{or } (x - 3x^{\frac{1}{2}}) - 2 = \pm\sqrt{24 - 5(x - 3x^{\frac{1}{2}})},$$

$$\therefore (x - 3x^{\frac{1}{2}})^2 - 4(x - 3x^{\frac{1}{2}}) + 4 = 24 - 5.(x - 3x^{\frac{1}{2}})$$

$$(x - 3x^{\frac{1}{2}})^2 + (x - 3x^{\frac{1}{2}}) = 20,$$

$$(x - 3x^{\frac{1}{2}})^2 + (x - 3x^{\frac{1}{2}}) + \frac{1}{4} = \frac{81}{4},$$

$$\therefore (x - 3x^{\frac{1}{2}}) + \frac{1}{2} = \pm\frac{9}{2}$$

$$\text{and } x - 3x^{\frac{1}{2}} = 4 \text{ or } -5,$$

$$\text{Again, } x - 3x^{\frac{1}{2}} + \frac{9}{4} = \frac{25}{4} \text{ or } -\frac{11}{4},$$

$$\therefore\ x^{\frac{1}{2}} - \frac{3}{2} = \pm\frac{5}{2} \text{ or } \pm\frac{\sqrt{-11}}{2};$$

$$x^{\frac{1}{2}} = 4 \text{ or } -1 \text{ or } \frac{3 \pm \sqrt{-11}}{2}$$

$$\text{and } x = 16 \text{ or } 1 \text{ or } \frac{9 \pm 6\sqrt{-11} - 11}{4}$$

$$\text{or } x = 16 \text{ or } 1 \text{ or } \frac{\pm 3\sqrt{-11} - 1}{2}$$

79. Given $\overline{4x+1}|^2 + 4x^{\frac{1}{2}}(4x+1) = 1912 - (10x + 3x^{\frac{1}{2}})$, to find the values of x.

$$\overline{4x+1}|^2 + 4x^{\frac{1}{2}}(4x+1) + 4x = 1912 - (6x + 3x^{\frac{1}{2}}),$$

$$\therefore\ 4x + 1 + 2x^{\frac{1}{2}} = \sqrt{1912 - 3.(2x + x^{\frac{1}{2}})}$$

$$\text{or } 2.(2x+x^{\frac{1}{2}}) + 1 = \sqrt{1912-3.(2x+x^{\frac{1}{2}})}$$

$$\therefore\ 4(2x+x^{\frac{1}{2}})^2 + 4(2x+x^{\frac{1}{2}}) + 1 = 1912-3.(2x+x^{\frac{1}{2}}),$$

$$\text{and } \overline{2x+x^{\frac{1}{2}}}\big|^2 + \frac{7}{4}(2x+x^{\frac{1}{2}}) = \frac{1911}{4},$$

$$\overline{2x+x^{\frac{1}{2}}}\big|^2 + \frac{7}{4}(2x+x^{\frac{1}{2}}) + \frac{49}{64} = \frac{1911}{4} + \frac{49}{64} = \frac{30625}{64},$$

$$\therefore\ 2x + x^{\frac{1}{2}} + \frac{7}{8} = \pm\frac{175}{8}$$

$$\text{and } 2x + x^{\frac{1}{2}} = 21 \text{ or } -\frac{182}{8},$$

$$\text{or } x + \frac{x^{\frac{1}{2}}}{2} = \frac{21}{2} \text{ or } -\frac{91}{8}$$

$$\text{Again } x + \frac{x^{\frac{1}{2}}}{2} + \frac{1}{16} = \frac{168+1}{16} \text{ or } \frac{1-182}{16},$$

$$= \frac{169}{16} \text{ or } \frac{-181}{16},$$

$$\therefore\ x^{\frac{1}{2}} + \frac{1}{4} = \pm\frac{13}{4} \text{ or } \frac{\pm\sqrt{-181}}{4},$$

$$x^{\frac{1}{2}} = 3 \text{ or } -\frac{7}{2} \text{ or } \frac{-1\pm\sqrt{-181}}{4}$$

$$\text{and } x = 9 \text{ or } \frac{49}{4} \text{ or } \frac{-90\mp\sqrt{-181}}{8}.$$

80. Given $8x^2 - 13 = \frac{3x}{2} + \sqrt{6x^3+52x^2}$, to find the values of x.

Multiplying by 4 we have,

$$32x^2 - 52 = 6x + 4x\sqrt{6x+52},$$

$$\text{or } 6x + 52 + 4x\sqrt{6x+52} + 4x^2 = 32x^2 + 4x^2 = 36x^2,$$

$$\sqrt{6x+52} + 2x = \pm 6x,$$

$$\sqrt{6x+52} = 4x \text{ or } -8x,$$

$$6x+52 = 16x^2 \text{ or } 64x^2;$$

$\therefore 16x^2 - 6x = 52$ or $64x^2 - 6x = 52$,

$x^2 - \frac{3x}{8} = \frac{13}{4}$ $\qquad$ $x^2 - \frac{3x}{32} = \frac{13}{16}$

$x^2 - \frac{3x}{8} + \frac{9}{256} = \frac{841}{256}$ $\qquad$ $x^2 - \frac{3x}{32} + \frac{9}{4096} = \frac{3337}{4096}$

$x - \frac{3}{16} = \pm \frac{29}{16}$ $\qquad$ $x - \frac{3}{64} = \frac{\pm\sqrt{3337}}{64}$

$$x = 2 \text{ or } -\frac{13}{8} \text{ or } \frac{3 \pm \sqrt{3337}}{64}.$$

81. Given $4x^2 + 21x + 8x^{\frac{1}{2}}\sqrt{7x^2 - 5x} = 207 - \frac{4x^2}{3}$,

to find the values of x.

Here $\frac{16x^2}{3} + 21x + 8x\sqrt{7x - 5} = 207$,

$$\frac{16x^2}{9} + 7x + \frac{8x}{3}\sqrt{7x - 5} = 69,$$

$$7x - 5 + \frac{8x}{3}\sqrt{7x - 5} + \frac{16x^2}{9} = 64$$

$$\sqrt{7x - 5} + \frac{4x}{3} = \pm 8,$$

$$3\sqrt{7x - 5} = \pm 24 - 4x,$$

$$63x - 45 = 576 \mp 192x + 16x^2;$$

$\therefore 16x^2 - 255x = -621$, $\qquad$ or $16x^2 + 129x = -621$

$x^2 - \frac{255}{16} + \frac{65025}{1024} = \frac{65025}{1024} - \frac{621}{16}$ $\qquad$ $x^2 + \frac{129}{16}x + \left(\frac{129}{32}\right)^2$

$\qquad = \frac{16641}{1024} - \frac{621}{16}$

$= \frac{65025 - 39744}{1024}$ $\qquad$ $= \frac{16641 - 39744}{1024}$,

$= \frac{25281}{1024}$ $\qquad$ $= \frac{-23103}{1024}$;

$$x - \frac{255}{32} = \pm \frac{159}{32} \qquad x + \frac{129}{32} = \frac{\pm\sqrt{-23103}}{32}$$

$$x = \frac{96}{32} \text{ or } \frac{414}{32} \qquad = \pm\frac{3\sqrt{-2567}}{32},$$

$$x = 3 \text{ or } \frac{207}{16} \qquad \text{or } \frac{-129 \pm 3\sqrt{-2567}}{32}.$$

82. Given $\dfrac{2x+\sqrt{x}}{2x-\sqrt{x}} = 3\dfrac{7}{15} - 3 . \dfrac{2x-\sqrt{x}}{2x+\sqrt{x}}$, to find the values of x.

$$\frac{2\sqrt{x}+1}{2\sqrt{x}-1} + 3 . \frac{2\sqrt{x}-1}{2\sqrt{x}+1} = \frac{52}{15},$$

$$\frac{4x+4\sqrt{x}+1+12x-12\sqrt{x}+3}{4x-1} = \frac{52}{15},$$

$$\frac{16x-8\sqrt{x}+4}{4x-1} = \frac{52}{15}$$

$$\frac{4x-2\sqrt{x}+1}{4x-1} = \frac{13}{15}$$

$$60x - 30\sqrt{x} + 15 = 52x - 13,$$

$$8x - 30\sqrt{x} = -28,$$

$$x - \frac{15}{4}\sqrt{x} = -\frac{14}{4},$$

$$x - \frac{15}{4}\sqrt{x} + \frac{225}{64} = \frac{225}{64} - \frac{14}{4} = \frac{1}{64}.$$

$$\sqrt{x} - \frac{15}{8} = \pm\frac{1}{8},$$

$$\sqrt{x} = 2 \text{ or } \frac{7}{4},$$

$$x = 4 \text{ or } \frac{49}{16}.$$

$$\sqrt{y} - \frac{1}{4} = \pm\frac{7}{4} \text{ or } \frac{\pm\sqrt{241}}{4}$$

$$\sqrt{y} = 2 \text{ or } -\frac{3}{2} \text{ or } \frac{1 \pm \sqrt{241}}{4}$$

$$y = 4 \text{ or } \frac{9}{4} \text{ or } \frac{1 \pm 2\sqrt{241} + 241}{16}$$

$$= 4 \text{ or } \frac{9}{4} \text{ or } \frac{242 \pm 2\sqrt{241}}{16}$$

$$= 4 \text{ or } \frac{9}{4} \text{ or } \frac{121 \pm \sqrt{241}}{8}$$

$$\text{and } x = 5 \text{ or } \frac{17}{2} \text{ or } \frac{-69 \mp \sqrt{241}}{4}.$$

11. Given $\left.\begin{array}{l} \dfrac{x^4}{y^2} + \dfrac{2x^3}{y} = 9\dfrac{39}{49} \\ \text{and } x + y^2 = 65 \end{array}\right\}$ to find the values of x and y.

$$\frac{x^4}{y^2} + \frac{2x^2}{y} = \frac{480}{49},$$

$$\frac{x^4}{y^2} + \frac{2x^2}{y} + 1 = \frac{480}{49} + 1 = \frac{529}{49},$$

$$\frac{x^2}{y} + 1 = \pm\frac{23}{7},$$

$$\frac{x^2}{y} = \pm\frac{23}{7} - 1 = \frac{16}{7} \text{ or } -\frac{30}{7},$$

$$x^2 = \frac{16y}{7} \text{ or } -\frac{30y}{7},$$

$$y^2 + \frac{16y}{7} + \frac{64}{49} = 65 + \frac{64}{49} = \frac{3249}{49},$$

$$y + \frac{8}{7} = \pm\frac{57}{7},$$

$$y = 7 \text{ or } \frac{-65}{7},$$

9. Given $\overline{x+y}|^2 - 3y = 28 + 3x$ and $2xy + 3x = 35$ } to find the values of x and y.

$$\overline{x+y}|^2 - 3\,.\,\overline{x+y} + \frac{9}{4} = 28 + \frac{9}{4} = \frac{121}{4}$$

$$\therefore\ x + y - \frac{3}{2} = \pm\frac{11}{2}$$

$$x + y = 7 \text{ or } -4$$

$$y = 7 - x \text{ or } -x - 4;$$

$$\therefore\ 14x - 2x^2 + 3x = 35 \qquad \text{or } -2x^2 - 8x + 3x = 35$$

$$2x^2 - 17x = -35 \qquad 2x^2 + 5x = -35$$

$$x^2 - \frac{17x}{2} + \frac{289}{16} = \frac{9}{16} \qquad x^2 + \frac{5x}{2} + \frac{25}{16} = -\frac{255}{16}$$

$$x - \frac{17}{4} = \pm\frac{3}{4} \qquad x + \frac{5}{4} = \frac{\pm\sqrt{-255}}{4}$$

$$x = 5 \text{ or } \frac{7}{2} \qquad x = \frac{-5 \pm \sqrt{-255}}{4}$$

$\therefore\ x = 5$ or $\frac{7}{2}$ or $\frac{-5 \pm \sqrt{-255}}{4}$; and $y = 2$ or $\frac{7}{2}$ or $\frac{-11 \mp \sqrt{-255}}{4}$.

10. Given $x^2 + 10x + y = 119 - 2\sqrt{y}\,.\times(x+5)$ and $x + 2y = 13$ } to find the values of x and y.

$$x^2 + 2x\sqrt{y} + y + 10(x + \sqrt{y}) + 25 = 144,$$

$$x + \sqrt{y} + 5 = \pm 12,$$

$$\therefore\ x + \sqrt{y} = 7 \text{ or } -17$$

$$\text{but } x + 2y = 13$$

$$\therefore\ 2y - \sqrt{y} = 6 \text{ or } 30$$

$$y - \frac{\sqrt{y}}{2} = 3 \text{ or } 15$$

$$y - \frac{\sqrt{y}}{2} + \frac{1}{16} = \frac{49}{16} \text{ or } \frac{241}{16}$$

$$\sqrt{y} - \frac{1}{4} = \pm\frac{7}{4} \text{ or } \frac{\pm\sqrt{241}}{4}$$

$$\sqrt{y} = 2 \text{ or } -\frac{3}{2} \text{ or } \frac{1 \pm \sqrt{241}}{4}$$

$$y = 4 \text{ or } \frac{9}{4} \text{ or } \frac{1 \pm 2\sqrt{241} + 241}{16}$$

$$= 4 \text{ or } \frac{9}{4} \text{ or } \frac{242 \pm 2\sqrt{241}}{16}$$

$$= 4 \text{ or } \frac{9}{4} \text{ or } \frac{121 \pm \sqrt{241}}{8}$$

$$\text{and } x = 5 \text{ or } \frac{17}{2} \text{ or } \frac{-69 \mp \sqrt{241}}{4}.$$

11. Given $\left.\begin{array}{l} \dfrac{x^4}{y^2} + \dfrac{2x^2}{y} = 9\dfrac{39}{49} \\ \text{and } x + y^2 = 65 \end{array}\right\}$ to find the values of x and y.

$$\frac{x^4}{y^2} + \frac{2x^2}{y} = \frac{480}{49},$$

$$\frac{x^4}{y^2} + \frac{2x^2}{y} + 1 = \frac{480}{49} + 1 = \frac{529}{49},$$

$$\frac{x^2}{y} + 1 = \pm\frac{23}{7},$$

$$\frac{x^2}{y} = \pm\frac{23}{7} - 1 = \frac{16}{7} \text{ or } -\frac{30}{7},$$

$$x^2 = \frac{16y}{7} \text{ or } -\frac{30y}{7},$$

$$y^2 + \frac{16y}{7} + \frac{64}{49} = 65 + \frac{64}{49} = \frac{3249}{49},$$

$$y + \frac{8}{7} = \pm\frac{57}{7},$$

$$y = 7 \text{ or } \frac{-65}{7},$$

Again, $y^2 - \frac{30y}{7} = 65,$

$$y^2 - \frac{30y}{7} + \frac{225}{49} = 65 + \frac{225}{49},$$

$$= \frac{3410}{49},$$

$$y - \frac{15}{7} = \frac{\pm\sqrt{3410}}{7},$$

$$y = \frac{15 \pm \sqrt{3410}}{7},$$

$$\therefore\ x = \pm\, 4 \text{ or } \frac{\pm\, 4\sqrt{-65}}{7} \text{ or } \frac{\pm\sqrt{-450 \mp 30\sqrt{3410}}}{\sqrt{7}}.$$

12. Given $\left.\begin{array}{r} x + y + \sqrt{x + y} = 6 \\ \text{and } x^2 + y^2 = 10 \end{array}\right\}$ to find the values of x and y.

$$x + y + \sqrt{x + y} + \frac{1}{4} = \frac{25}{4},$$

$$\sqrt{x + y} + \frac{1}{2} = \pm\frac{5}{2},$$

$$\sqrt{x + y} = 2 \text{ or } -3,$$

$$x + y = 4 \text{ or } 9,$$

$$\therefore\ x^2 + 2xy + y^2 = 16 \text{ or } 81,$$

$$\text{but } x^2 + y^2 = 10,$$

$$\therefore\ 2xy = 6 \text{ or } 71,$$

$$x^2 - 2xy + y^2 = 4 \text{ or } -61.$$

$$x - y = \pm\, 2 \text{ or } \pm\sqrt{-61},$$

$$x + y = 4 \text{ or } 9.$$

$$2x = 6 \text{ or } 2 \text{ or } 9 \pm \sqrt{-61},$$

$$x = 3 \text{ or } 1 \text{ or } \frac{9 \pm \sqrt{-61}}{2},$$

$$y = 1 \text{ or } 3 \text{ or } \frac{9 \mp \sqrt{-61}}{2},$$

13. Given $\left.\begin{array}{l} x^2 + 4\sqrt{x^2+3y+5} = 55 - 3y \\ \text{and } 6x - 7y = 16 \end{array}\right\}$ to find the values of x and y.

$$x^2 + 3y + 5 + 4\sqrt{x^2+3y+5} + 4 = 64,$$

$$\therefore \sqrt{x^2+3y+5} + 2 = \pm 8,$$

$$\text{and } \sqrt{x^2+3y+5} = 6 \text{ or } -10,$$

$$x^2 + 3y + 5 = 36 \text{ or } 100,$$

$$x^2 + 3y = 31 \text{ or } 95,$$

$$7x^2 + 21y = 217 \text{ or } 665,$$

$$18x - 21y = 48,$$

$$\therefore 7x^2 + 18x = 265 \text{ or } 713,$$

$$x^2 + \frac{18x}{7} + \frac{81}{49} = \frac{265}{7} + \frac{81}{49} \text{ or } \frac{713}{7} + \frac{81}{49},$$

$$= \frac{1936}{49} \text{ or } \frac{5072}{49},$$

$$x + \frac{9}{7} = \pm\frac{44}{7} \text{ or } \frac{\pm\sqrt{5072}}{7},$$

$$x = 5 \text{ or } \frac{-53}{7} \text{ or } \frac{-9 \pm 4\sqrt{317}}{7},$$

$$\therefore y = 2 \text{ or } \frac{430}{49} \text{ or } \frac{-166 \mp 24\sqrt{317}}{49}.$$

14. Given $\left.\begin{array}{l} x^2 + 3x + y = 73 - 2xy \\ \text{and } y^2 + 3y + x = 44 \end{array}\right\}$ to find the values of x and y.

$$x^2 + 2xy + y^2 + 4x + 4y = 117,$$

$$\overline{x+y}^2 + 4.\overline{x+y} + 4 = 121,$$

$$x + y + 2 = \pm 11,$$

$$x + y = 9 \text{ or } -13,$$

$$\text{but } y^2 + x + 3y = 44,$$

$$\therefore y^2 + 2y = 35 \text{ or } 57,$$

$$y^2 + 2y + 1 = 36 \text{ or } 58,$$

$$\text{or } 2.(2x+x^{\frac{1}{2}}) + 1 = \sqrt{1912-3.(2x+x^{\frac{1}{2}})}$$

$$\therefore\ 4(2x+x^{\frac{1}{2}})^2 + 4(2x+x^{\frac{1}{2}}) + 1 = 1912-3.(2x+x^{\frac{1}{2}}),$$

$$\text{and } \overline{2x+x^{\frac{1}{2}}}\Big|^2 + \frac{7}{4}(2x+x^{\frac{1}{2}}) = \frac{1911}{4},$$

$$\overline{2x+x^{\frac{1}{2}}}\Big|^2 + \frac{7}{4}(2x+x^{\frac{1}{2}}) + \frac{49}{64} = \frac{1911}{4} + \frac{49}{64} = \frac{30625}{64},$$

$$\therefore\ 2x + x^{\frac{1}{2}} + \frac{7}{8} = \pm\frac{175}{8}$$

$$\text{and } 2x + x^{\frac{1}{2}} = 21 \text{ or } -\frac{182}{8},$$

$$\text{or } x + \frac{x^{\frac{1}{2}}}{2} = \frac{21}{2} \text{ or } -\frac{91}{8}$$

$$\text{Again } x + \frac{x^{\frac{1}{2}}}{2} + \frac{1}{16} = \frac{168+1}{16} \text{ or } \frac{1-182}{16},$$

$$= \frac{169}{16} \text{ or } \frac{-181}{16},$$

$$\therefore\ x^{\frac{1}{2}} + \frac{1}{4} = \pm\frac{13}{4} \text{ or } \frac{\pm\sqrt{-181}}{4},$$

$$x^{\frac{1}{2}} = 3 \text{ or } -\frac{7}{2} \text{ or } \frac{-1\pm\sqrt{-181}}{4}$$

$$\text{and } x = 9 \text{ or } \frac{49}{4} \text{ or } \frac{-90\mp\sqrt{-181}}{8}.$$

80. Given $8x^2 - 13 = \frac{3x}{2} + \sqrt{6x^3+52x^2}$, to find the values of x.

Multiplying by 4 we have,

$$32x^2 - 52 = 6x + 4x\sqrt{6x+52},$$

$$\text{or } 6x + 52 + 4x\sqrt{6x+52} + 4x^2 = 32x^2 + 4x^2 = 36x^2,$$

$$\sqrt{6x+52} + 2x = \pm 6x,$$

$$\sqrt{6x+52} = 4x \text{ or } -8x,$$

$$6x+52 = 16x^2 \text{ or } 64x^2;$$

$\therefore\ 16x^2 - 6x = 52$ or $64x^2 - 6x = 52,$

$x^2 - \frac{3x}{8} = \frac{13}{4}$ $\qquad x^2 - \frac{3x}{32} = \frac{13}{16}$

$x^2 - \frac{3x}{8} + \frac{9}{256} = \frac{841}{256}$ $\qquad x^2 - \frac{3x}{32} + \frac{9}{4096} = \frac{3337}{4096}$

$x - \frac{3}{16} = \pm\frac{29}{16}$ $\qquad x - \frac{3}{64} = \frac{\pm\sqrt{3337}}{64}$

$$x = 2 \text{ or } -\frac{13}{8} \text{ or } \frac{3 \pm \sqrt{3337}}{64}.$$

81. Given $4x^2 + 21x + 8x^{\frac{1}{2}}\sqrt{7x^2 - 5x} = 207 - \frac{4x^2}{3}$, to find the values of x.

Here $\frac{16x^2}{3} + 21x + 8x\sqrt{7x - 5} = 207,$

$$\frac{16x^2}{9} + 7x + \frac{8x}{3}\sqrt{7x - 5} = 69,$$

$$7x - 5 + \frac{8x}{3}\sqrt{7x - 5} + \frac{16x^2}{9} = 64$$

$$\sqrt{7x - 5} + \frac{4x}{3} = \pm 8,$$

$$3\sqrt{7x - 5} = \pm 24 - 4x,$$

$$63x - 45 = 576 \mp 192x + 16x^2;$$

$\therefore\ 16x^2 - 255x = -621,$ or $16x^2 + 129x = -621$

$x^2 - \frac{255}{16} + \frac{65025}{1024} = \frac{65025}{1024} - \frac{621}{16}$ $\qquad x^2 + \frac{129}{16}x + \left(\frac{129}{32}\right)^2 = \frac{16641}{1024} - \frac{621}{16}$

$= \frac{65025 - 39744}{1024}$ $\qquad = \frac{16641 - 39744}{1024},$

$= \frac{25281}{1024}$ $\qquad = \frac{-23103}{1024};$

17. Given $\left.\begin{aligned} x^4 + y^4 &= 97 \\ \text{and } x + y &= 5 \end{aligned}\right\}$ to find the values of x and y.

$$\begin{aligned} x^4 + 4x^3y + 6x^2y^2 + 4xy^3 + y^4 &= 625 \\ x^4 \qquad\qquad\qquad\qquad + y^4 &= 97, \end{aligned}$$

$$\therefore\ 4x^3y + 6x^2y^2 + 4xy^3 = 528$$

$$\text{or } 2x^3y + 3x^2y^2 + 2xy^3 = 264$$

$$\therefore\ 2x^2 + 3xy + 2y^2 = \frac{264}{xy}$$

$$\text{but } 2x^2 + 4xy + 2y^2 = 50$$

$$xy = 50 - \frac{264}{xy},$$

$$\begin{aligned} x^2y^2 - 50xy &= -264, \\ x^2y^2 - 50xy + 625 &= 625 - 264 = 361, \\ xy - 25 &= \pm 19, \\ xy &= 44 \text{ or } 6, \\ x^2 + 2xy + y^2 &= 25. \\ 4xy &= 24 \text{ or } 176, \end{aligned}$$

$$\begin{aligned} x^2 - 2xy + y^2 &= 1 \text{ or } -151, \\ \therefore\ x - y &= \pm 1 \text{ or } \pm\sqrt{-151}, \\ \text{but } x + y &= 5, \end{aligned}$$

consequently $x = 3$ or 2 or $\dfrac{5 \pm \sqrt{-151}}{2}$,

and $y = 2$ or 3 or $\dfrac{5 \mp \sqrt{-151}}{2}$.

18. Given $\left.\begin{aligned} \sqrt{\frac{3x-2y}{2x}} + \sqrt{\frac{2x}{3x-2y}} &= 2 \\ \text{and } x^2 - 18 &= x(4y - 9) \end{aligned}\right\}$ to find the values of x and y.

Multiplying both sides of the first equation by $\sqrt{\dfrac{3x-2y}{2x}}$,

we have $\dfrac{3x-2y}{2x} + 1 = 2 .\sqrt{\dfrac{3x-2y}{2x}}$

$$\text{or } \frac{3x-2y}{2x} - 2.\sqrt{\frac{3x-2y}{2x}} + 1 = 0,$$

$$\sqrt{\frac{3x-2y}{2x}} - 1 = 0,$$

$$\frac{3x-2y}{2x} = 1,$$

$$\therefore\ 3x - 2y = 2x,$$

$$x = 2y,$$

$$\text{and } y = \frac{x}{2};$$

Again, $x^2 - 18 = x(2x - 9) = 2x - 9x$,

$$x^2 - 9x = -18,$$

$$x^2 - 9x + \frac{81}{4} = \frac{81-72}{4} = \frac{9}{4},$$

$$x - \frac{9}{2} = \pm\frac{3}{2},$$

$$x = 6 \text{ or } 3,$$

$$y = \frac{x}{2} = 3 \text{ or } \frac{3}{2}$$

19. Given $\left.\begin{array}{l} x+4\sqrt{x}+4y=21+8\sqrt{y}+4\sqrt{xy} \\ \text{and } \sqrt{x}+\sqrt{y}=6 \end{array}\right\}$ to find the values of x and y.

$$x - 4\sqrt{xy} + 4y + 4\sqrt{x} - 8\sqrt{y} = 21,$$

$$\text{or } \overline{\sqrt{x} - 2\sqrt{y}}\,^2 + 4(\sqrt{x} - 2\sqrt{y}) + 4 = 25,$$

$$\sqrt{x} - 2\sqrt{y} + 2 = \pm 5,$$

$$\therefore\ \sqrt{x} - 2\sqrt{y} = 3 \text{ or } -7;$$

$$\text{but } \sqrt{x} + \sqrt{y} = 6,$$

$$3\sqrt{y} = 3 \text{ or } 13,$$

$$\sqrt{y} = 1 \text{ or } \frac{13}{3},$$

$$y = 1 \text{ or } \frac{169}{9},$$

$$\sqrt{x} = 6 - \sqrt{y} = 5 \text{ or } \frac{5}{3},$$

$$x = 25 \text{ or } \frac{25}{9}.$$

20. Given $\left.\begin{array}{l} x + y = 5 \\ \text{and } (x^2 + y^2) \times (x^3 + y^3) = 455 \end{array}\right\}$ to find the values of x and y.

$$x^5 + 5x^4y + 10x^3y^2 + 10x^2y^3 + 5xy^4 + y^5 = 3125$$
$$x^5 + x^3y^2 + x^2y^3 + y^5 = 455$$

$$5x^4y + 9x^3y^2 + 9x^2y^3 + 5xy^4 = 2670$$

$$5x^3 + 9x^2y + 9xy^2 + 5y^3 = \frac{2670}{xy}$$

$$\text{but } 5x^3 + 15x^2y + 15xy^2 + 5y^3 = 625$$

$$\therefore\ 6x^2y + 6xy^2 = 625 - \frac{2670}{xy}$$

$$\text{or } 6xy\,(x + y) = 625 - \frac{2670}{xy},$$

$$\text{and } 6xy = 125 - \frac{534}{xy};$$

$$\therefore\ 6x^2y^2 - 125xy = -534,$$

$$x^2y^2 - \frac{125}{6}xy + \frac{15625}{144} = \frac{15625}{144} - \frac{534}{6}$$

$$= \frac{15625 - 12816}{144},$$

$$= \frac{2809}{144},$$

$$xy - \frac{125}{12} = \pm\frac{53}{12}$$

$$xy = \frac{178}{12} \text{ or } \frac{72}{12},$$

$$= \frac{89}{6} \text{ or } 6,$$

$$x^2 + 2xy + y^2 = 25,$$
$$4xy \quad = 24 \text{ or } \frac{178}{3}.$$

$$x^2 - 2xy + y^2 = 1 \text{ or } -\frac{103}{3}$$

$$\therefore\ x - y = \pm 1 \text{ or } \pm\sqrt{-\frac{103}{3}}$$

$$\text{but } x + y = 5.$$

$$x = 3 \text{ or } 2 \text{ or } \frac{5}{2} \pm \frac{1}{2}\sqrt{-\frac{103}{3}}$$

$$x = 2 \text{ or } 3 \text{ or } \frac{5}{2} \mp \frac{1}{2}\sqrt{-\frac{103}{3}}.$$

21. Given $x + y - \sqrt{\dfrac{x+y}{x-y}} = \dfrac{6}{x-y}$ and $x^2 + y^2 = 41$ } to find the values of x and y.

Here $x^2 - y^2 - \sqrt{x^2 - y^2} = 6,$

$$x^2 - y^2 - \sqrt{x^2 - y^2} + \frac{1}{4} = \frac{25}{4}$$

$$\sqrt{x^2 - y^2} - \frac{1}{2} = \pm\frac{5}{2}$$

$$\sqrt{x^2 - y^2} = 3 \text{ or } -2,$$

$$\therefore\ x^2 - y^2 = 9 \text{ or } 4$$

$$\text{but } x^2 + y^2 = 41.$$

$$x^2 = 25 \text{ or } \frac{45}{2},$$

$$x = \pm 5 \pm 3\sqrt{\frac{5}{2}},$$

$$y^2 = 16 \text{ or } \frac{37}{2},$$

$$y = \pm 4 \text{ or } \pm\sqrt{\frac{37}{2}}.$$

22. Given $\left.\begin{array}{l}\dfrac{x^4}{y^2} + \dfrac{y^4}{x^2} = 136\,\dfrac{1}{9} - 2xy \\ \text{and } x + 4 = 14 - y\end{array}\right\}$ to find the values of x and y.

Here $\dfrac{x^4}{y^2} + 2xy + \dfrac{y^4}{x^2} = \dfrac{1225}{9}$

$$\therefore \frac{x^2}{y} + \frac{y^2}{x} = \pm\frac{35}{3}$$

$$\text{or } x^3 + y^3 = \pm\frac{35xy}{3},$$

$$\text{but } x + y = 10;$$

$$\therefore x^3 + 3x^2y + 3xy^2 + y^3 = 1000,$$

$$\text{and } x^3 \qquad\qquad + y^3 = \pm\frac{35xy}{3},$$

$$\therefore 3xy.\overline{x+y} = 1000 \mp \frac{35xy}{3}$$

$$30xy = 1000 \mp \frac{35xy}{3},$$

$$90xy \pm 35xy = 3000,$$

$$125xy \text{ or } 55xy = 3000$$

$$xy = \frac{3000}{125} \text{ or } \frac{3000}{55}$$

$$= 24 \text{ or } \frac{600}{11}.$$

Now $x^2 + 2xy + y^2 = 100$

and $4xy \qquad = 96$ or $\dfrac{2400}{11}$,

$$\therefore x^2 - 2xy + y^2 = 4 \text{ or } -\frac{1300}{11}$$

$$x - y = \pm 2 \text{ or } \pm 10.\sqrt{-\frac{13}{11}}$$

$$x + y = 10.$$

$$x = 6 \text{ or } 4 \text{ or } 5 \pm 5.\sqrt{-\frac{13}{11}}$$

$$y = 4 \text{ or } 6 \text{ or } 5 \mp 5.\sqrt{-\frac{13}{11}}.$$

23. Given $\left.\begin{aligned} \frac{x+y}{x-y} - \frac{x-y}{x+y} &= 4\,\frac{4}{5} \\ \text{and } \sqrt{\frac{x+y}{x^4}} + \frac{1}{x} &= \frac{4}{9\sqrt{x-y}} \end{aligned}\right\}$ to find the values of x and y.

From the first equation,

$$\frac{x^2 + 2xy + y^2 - x^2 + 2xy - y^2}{x^2 - y^2} = \frac{24}{5};$$

$$\text{or } \frac{4xy}{x^2 - y^2} = \frac{24}{5}.$$

$$\text{Again, } \frac{x-y}{x^2} + \frac{\sqrt{x-y}}{x} = \frac{4}{9},$$

$$\frac{x-y}{x^2} + \frac{\sqrt{x-y}}{x} + \frac{1}{4} = \frac{1}{4} + \frac{4}{9} = \frac{25}{36};$$

$$\therefore \frac{\sqrt{x-y}}{x} + \frac{1}{2} = \pm\frac{5}{6},$$

$$\text{and } \frac{\sqrt{x-y}}{x} = \frac{1}{3} \text{ or } -\frac{4}{3};$$

$$\therefore \frac{x-y}{x^2} = \frac{1}{9} \text{ or } \frac{16}{9}.$$

Multiplying this equation by $\dfrac{4xy}{x^2 - y^2} = \dfrac{24}{5}$

$$\frac{y}{x.\overline{x+y}} = \frac{2}{15} \text{ or } \frac{32}{15};$$

$$\text{First, } \frac{9-x}{18x - x^2} = \frac{2}{15},$$

$$\text{or } 135 - 15x = 36x - 2x^2,$$

$$2x^2 - 51x = -135;$$

$$\therefore x^2 - \frac{51}{2}x + \overline{\frac{51}{4}}^2 = -\frac{135}{2} + \frac{2601}{16} = \frac{1521}{16};$$

$$\therefore x - \frac{51}{4} = \pm\frac{39}{4}.$$

$$\text{and } x = \quad \text{or } \frac{45}{2}$$

$$\text{Again, } \frac{9-16x}{15x-16x^2} = \frac{32}{15}$$

$$135 - 240x = 576x - 512x^2;$$

$$\therefore 512x^2 - 816x = -135;$$

$$x^2 - \frac{51}{32}x = -\frac{135}{512};$$

$$\therefore x^2 - \frac{51}{32}x + \frac{2601}{4096} = \frac{2601}{4096} - \frac{135}{512} = \frac{2601-1080}{4096} = \frac{1521}{4096};$$

$$\therefore x - \frac{51}{64} = \pm\frac{39}{64}$$

$$\text{and } x = \frac{3}{16} \text{ or } \frac{45}{32}.$$

$$\therefore x = 3 \text{ or } \frac{45}{2}, \text{ or } \frac{3}{16} \text{ or } \frac{45}{32}$$

$$y = 2 \text{ or } -\frac{135}{4}, \text{ or } \frac{1}{6} \text{ or } -\frac{135}{32}.$$

24 Given $\sqrt{6\sqrt{x} - 4\sqrt{y}} - \frac{1}{2}\sqrt{x} = 1 - \frac{1}{2}\sqrt{y}$ and $x - y = 32$ [illegible]

to find the values of x and y.

From the first equation,

$$\sqrt{x} - \sqrt{y} - 2\sqrt{6\sqrt{x} - 4\sqrt{y}} = [illegible];$$

$$\therefore 6\sqrt{x} - 4\sqrt{y} - 12\sqrt{6\sqrt{x} - 4\sqrt{y}} - [illegible] = [illegible] = [illegible]$$

$$\text{and } \sqrt{6\sqrt{x} - 4\sqrt{y}} - 6 = \pm [illegible].$$

$\therefore \quad \sqrt{6(\sqrt{x}+\sqrt{y})} = 6 \text{ or } -18,$

$6(\sqrt{x}+\sqrt{y}) = 36 \text{ or } 324,$

$\sqrt{x}+\sqrt{y} = 6 \text{ or } 54,$

but $\sqrt{x}-\sqrt{y} = 2 \text{ or } \frac{2}{9};$

$\therefore \quad \sqrt{x} = 4 \text{ or } \frac{244}{9}$

and $\sqrt{y} = 2 \text{ or } \frac{242}{9};$

$\therefore x = 16, \text{ or } \frac{59536}{81}$

and $y = 4 \text{ or } \frac{58564}{81}.$

25. Given $\left.\begin{array}{l} y^4 - 432 = 12xy^2 \\ \text{and } y^2 = 12 + 2xy \end{array}\right\}$ to find the values of x and y.

From the first equation, $x = \frac{y^4 - 432}{12y^2},$

and from the second, $x = \frac{y^2 - 12}{2y};$

$\therefore \frac{y^4 - 432}{12y^2} = \frac{y^2 - 12}{2y},$

and $y^4 - 432 = 6y^3 - 72y,$

or $y^4 + 72y = 6y^3 + 432,$

or $y(y^3 + 72) = 6(y^3 + 72);$

$\therefore y = 6,$

and $x = \frac{37 - 12}{12} = 2.$

26. Given $\left.\begin{array}{l} \frac{4}{y^2} + \frac{4+y}{y} = \frac{8+4y}{x} + \frac{12y^2}{x^2} \\ \text{and } 4y^2 - xy = x \end{array}\right\}$ to find the values of x and y.

From the second equation $x = \dfrac{4y^2}{y+1}$.

Substituting this value in the first,

$$\frac{4}{y^2} + \frac{4+y}{y} = \frac{4.(y+2)\ (y+1)}{4y^2} + \frac{12y^2(y+1)^2}{16y^4}$$

$$= \frac{y^2+3y+2}{y^2} + \frac{3(y^2+2y+1)}{4y^2};$$

$$\text{or } \frac{4}{y^2} + \frac{4}{y} + 1 = 1 + \frac{3}{y} + \frac{2}{y^2} + \frac{3}{4} + \frac{3}{2y} + \frac{3}{4y^2};$$

$$\therefore \frac{5}{4y^2} - \frac{1}{2y} = \frac{3}{4}$$

$$\text{and } 3y^2 + 2y = 5;$$

$$\therefore y^2 + \frac{2y}{3} + \frac{1}{9} = \frac{5}{3} + \frac{1}{9} = \frac{16}{9},$$

$$y + \frac{1}{3} = \pm\frac{4}{3};$$

$$\therefore y = 1 \text{ or } -\frac{5}{3},$$

$$\text{and } x = \frac{4y^2}{y+1} = 2 \text{ or } -\frac{50}{3}.$$

27. Given $\left.\begin{array}{l}\sqrt{(1+x)^2+y^2} + \sqrt{(1-x)^2+y^2} = 4 \\ \text{and } (4-x^2)^2 = 18 - 4y^2,\end{array}\right\}$

to find the values of x and y.

Inverting both sides of the first equation,

$$\frac{1}{\sqrt{(1+x)^2+y^2} + \sqrt{(1-x)^2+y^2}} = \frac{1}{4}$$

Now multiplying numerator and denominator by

$\sqrt{(1+x)^2+y^2} - \sqrt{(1-x)^2+y^2}$, we have

$$\sqrt{(1+x)^2+y^2} - \sqrt{(1-x)^2+x^2} = x,$$

$$\text{but } \sqrt{(1+x)^2+y^2} + \sqrt{(1-x)^2+y^2} = 4,$$

$$\therefore\ 2\sqrt{(1+x)^2+y^2} = 4+x,$$

$$\text{and } 4+8x+4x^2+4y^2 = 16+8x+x^2,$$

$$\therefore\ 4y^2 = 12-3x^2 = 3(4-x^2)$$

substituting in the second equation,

$$\frac{16y^4}{9} = 18 - 4y^2.$$

$$\text{Then } y^4 + \frac{9y^2}{4} = \frac{81}{8},$$

$$\therefore\ y^4+\frac{9y^2}{4}+\frac{81}{64} = \frac{81+648}{64} = \frac{729}{64},$$

$$y^2+\frac{9}{8} = \pm\frac{27}{8},$$

$$y^2 = \frac{9}{4} \text{ or } -\frac{9}{2},$$

$$\therefore\ y = \pm\frac{3}{2} \text{ or } \pm 3\sqrt{-\frac{1}{2}},$$

$$\text{and } x = \pm 1 \text{ or } \pm\sqrt{10}.$$

28. Given
$$\left.\begin{aligned} \frac{x+\sqrt{x}+y}{x-\sqrt{x}+y} - \frac{\sqrt{x}-x-y}{\sqrt{x}+x+y} &= 2\frac{9}{40} \\ \text{and } y^2-\sqrt{xy^2} &= \frac{4x}{9} \end{aligned}\right\}$$

to find the values of x and y.

From the first equation,

$$\frac{\overline{x+y}+\sqrt{x}}{\overline{x+y}-\sqrt{x}} + \frac{\overline{x+y}-\sqrt{x}}{\overline{x+y}+\sqrt{x}} = 2\frac{9}{40},$$

$$\text{or } \frac{2.\overline{x+y}\big|^2+2x}{\overline{x+y}\big|^2-x} = 2\frac{9}{40},$$

$$\therefore\ 2.\overline{x+y}\big|^2+2x = 2\frac{9}{40}.\ \overline{x+y}\big|^2 - 2\frac{9}{40}x,$$

$$\therefore\ \frac{9}{40}\overline{x+y}\big|^2 = 4\frac{9}{40}x = \frac{169}{40}x,$$

$$\text{and } \overline{x+y}|^2 = \frac{169}{9}x,$$

$$\text{and } x + y = \pm \frac{13\sqrt{x}}{3}.$$

Now, from the second equation,

$$y^2 - y\sqrt{x} + \frac{x}{4} = \frac{x}{4} + \frac{4x}{9} = \frac{25x}{36},$$

$$y - \frac{\sqrt{x}}{2} = \pm \frac{5\sqrt{x}}{6},$$

$$y = \frac{4\sqrt{x}}{3} \text{ or } \frac{-\sqrt{x}}{3}.$$

$$\text{First, } x + \frac{4\sqrt{x}}{3} = \pm \frac{13\sqrt{x}}{3},$$

$$x = \frac{9\sqrt{x}}{3} \text{ or } -\frac{17\sqrt{x}}{3} = 3\sqrt{x} \text{ or } -\frac{17\sqrt{x}}{3},$$

$$\therefore \sqrt{x} = 3 \text{ or } -\frac{17}{3},$$

$$\text{and } x = 9 \text{ or } \frac{289}{9}.$$

$$\text{Again, } x - \frac{\sqrt{x}}{3} = \pm \frac{13\sqrt{x}}{3},$$

$$x = \frac{14\sqrt{x}}{3} \text{ or } -4\sqrt{x},$$

$$\sqrt{x} = \frac{14}{3} \text{ or } -4,$$

$$x = \frac{196}{9} \text{ or } 16,$$

$$\therefore x = 9 \text{ or } \frac{196}{9} \text{ or } \frac{289}{9} \text{ or } 16,$$

$$\text{and } y = 4 \text{ or } \frac{-14}{9} \text{ or } -\frac{68}{9} \text{ or } \frac{4}{3}.$$

29. Given
$$\left.\begin{aligned}\frac{x+\sqrt{x^2-y^2}}{x-\sqrt{x^2-y^2}} &= 4\,\frac{1}{4} - \frac{x-\sqrt{x^2-y^2}}{x+\sqrt{x^2-y^2}}\\ \text{and } x.\overline{x+y} &= 52 - \sqrt{x^2+xy+4}\end{aligned}\right\}$$
to find the values of x and y.

From the first equation, we have

$$\frac{x+\sqrt{x^2-y^2}}{x-\sqrt{x^2-y^2}} + \frac{x-\sqrt{x^2-y^2}}{x+\sqrt{x^2-y^2}} = \frac{17}{4}.$$

Reducing to a common denominator,

$$\frac{2x^2+2x^2-2y^2}{y^2} = \frac{17}{4},$$

$$\therefore\ 4x^2 = \frac{17}{4}y^2 + 2y^2 = \frac{25y^2}{4},$$

$$\therefore\ 4x = 5y.$$

Again, from the second equation,

$$x^2+xy+4+\sqrt{x^2+xy+4}+\frac{1}{4} = 52+\frac{17}{4} = \frac{225}{4},$$

$$\sqrt{x^2+xy+4}+\frac{1}{2} = \pm\frac{15}{2},$$

$$\sqrt{x^2+xy+4} = 7 \text{ or } -8,$$

$$\therefore\ x^2+xy+4 = 49 \text{ or } 64,$$

$$x^2+xy = 45 \text{ or } 60,$$

$$\text{but } y = \frac{4x}{5},$$

$$x^2+\frac{4x^2}{5} = 45 \text{ or } 60,$$

$$\text{or } \frac{9x^2}{5} = 45 \text{ or } 60,$$

$$\therefore\ 9x^2 = 225 \text{ or } 300,$$

$$x^2 = 25 \text{ or } \frac{100}{3},$$

$$\text{and } x = \pm 5 \text{ or } \pm\frac{10}{\sqrt{3}},$$

$$\therefore\ y = \frac{4x}{5} = \pm 4 \text{ or } \pm\frac{8}{\sqrt{3}}.$$

30. Given $5y + \dfrac{\sqrt{x^2 - 15y - 14}}{5} = \dfrac{x^2}{3} - 36$ and $\dfrac{x^2}{8y} + \dfrac{2x}{3} = \sqrt{\dfrac{x^2}{3y} + \dfrac{x^2}{4}} + \dfrac{y}{2}$ to find the values of x and y.

From the first equation,

$$x^2 - 15y - 108 = \frac{3}{5}\sqrt{x^2 - 15y - 14},$$

$$\therefore\ x^2 - 15y - 14 - \frac{3}{5}\sqrt{x^2 - 15y - 14} + \frac{9}{100} = \frac{9}{100} + 94$$

$$= \frac{9409}{100}.$$

Extracting the square root,

$$\sqrt{x^2 - 15y - 14} - \frac{3}{10} = \pm\frac{97}{10},$$

$$\therefore\ \sqrt{x^2 - 15y - 14} = 10 \text{ or } -\frac{47}{5}$$

$$x^2 - 15y - 14 = 100 \text{ or } \frac{2209}{25},$$

$$x^2 - 15y = 114 \text{ or } \frac{2559}{25}.$$

Again, from the second equation,

$$\frac{x^2}{8y} + \frac{2x}{3} + \frac{y}{2} = \sqrt{\frac{x^2}{3y} + \frac{x^2}{4}}.$$

Squaring both sides,

$$\frac{x^4}{64y^2} + \frac{4x^2}{9} + \frac{y^2}{4} + \frac{x^2}{6y} + \frac{x^2}{8} + \frac{2xy}{3} = \frac{x^3}{3y} + \frac{x^2}{4}$$

Hence $\dfrac{x^4}{64y^2} + \dfrac{4x^2}{9} + \dfrac{y^2}{4} - \dfrac{x^3}{6y} - \dfrac{x^2}{8} + \dfrac{2xy}{3} = 0,$

and extracting the square root,

$$\frac{x^2}{8y} - \frac{2x}{3} - \frac{y}{2} = 0,$$

$$\therefore\ x^2 - \frac{16xy}{3} = 4y^2,$$

$$x^2 - \frac{16xy}{3} + \frac{64y^2}{9} = \frac{(36+64)y^2}{9} = \frac{100y^2}{9}$$

$$x - \frac{8y}{3} = \pm\frac{10y}{3}$$

$$\text{and } x = 6y \text{ or } -\frac{2y}{3}$$

$$y = \frac{x}{6} \text{ or } -\frac{3x}{2}$$

$$\text{First, } x^2 - 15.\frac{x}{6} = 114 \text{ or } \frac{2539}{25}$$

$$\text{or } x^2 - \frac{5x}{2} + \frac{25}{16} = \frac{25}{16} + 114 = \frac{1849}{16} \text{ or } \frac{41569}{400}$$

$$\therefore\ x - \frac{5}{4} = \pm\frac{43}{4} \text{ or } \pm\frac{\sqrt{41569}}{20}$$

$$\text{and } x = 12 \text{ or } -\frac{19}{2} \text{ or } \frac{5}{4} \pm \frac{\sqrt{41569}}{20}$$

$$= 12 \text{ or } -\frac{19}{2} \text{ or } \frac{25 \pm \sqrt{41569}}{20}.$$

$$\text{Again, } x^2 - 15\left(-\frac{3x}{2}\right) = 114,$$

$$\text{or } x^2 + \frac{45x}{2} + \left(\frac{45}{4}\right)^2 = 114 + \frac{2025}{16} = \frac{3849}{16},$$

$$x + \frac{45}{4} = \pm\frac{\sqrt{3849}}{4},$$

$$\text{and } x = \frac{-45 \pm \sqrt{3849}}{4}.$$

$$\therefore\ x = 12 \text{ or } -\frac{19}{2} \text{ or } \frac{25 \pm \sqrt{41569}}{20} \text{ or } \frac{-45 \pm \sqrt{3849}}{4},$$

$$\text{and } y = 2 \text{ or } -\frac{19}{2} \text{ or } \frac{25 \pm \sqrt{41569}}{120} \text{ or } \frac{-135 \pm \sqrt{3849}}{8}.$$

31. Given $\left.\begin{aligned} &\sqrt{\frac{x+y^2}{4x}} + \frac{y}{\sqrt{y^2+x}} = \frac{y^2}{4}\sqrt{\frac{4x}{y^2+x}} \\ &\text{and } \frac{\sqrt{x}+\sqrt{x-y-1}}{\sqrt{x}-\sqrt{x-y-1}} = y+1 \end{aligned}\right\}$

to find the values of x and y.

Multiplying the first equation by $\sqrt{\frac{x+y^2}{4x}}$,

$$\frac{x+y^2}{4x} + \frac{y}{\sqrt{4x}} = \frac{y^2}{4},$$

$$\text{or } \frac{y^2}{x} + \frac{2y}{\sqrt{x}} + 1 = y^2,$$

$$\text{and } \frac{y}{\sqrt{x}} + 1 = \pm y,$$

$$\pm \sqrt{x}\, y - \sqrt{x} = y,$$

$$\sqrt{x} = \frac{y}{\pm y - 1}.$$

Multiplying numerator and denominator of the left side of the second equation by $\sqrt{x} + \sqrt{x-y-1}$,

$$\frac{2x - y - 1 + 2\sqrt{x}\sqrt{x-y-1}}{y+1} = y + 1,$$

and $2x - y - 1 + 2\sqrt{x}\sqrt{x-y-1} = (y+x)^2$ (1).

$$\text{Again, } \frac{\sqrt{x}-\sqrt{x-y-1}}{\sqrt{x}+\sqrt{x-y-1}} = \frac{1}{y+1},$$

and multiplying numerator and denominator as before by $\sqrt{x} - \sqrt{x-y-1}$, we have

$$2x - y - 1 - 2\sqrt{x}\sqrt{x-y-1} = 1 \quad (2).$$

adding together equations (1) and (2)

$$4x - 2y - 2 = 1 + \overline{y+1}|^2$$

$$\text{and } 4x = y^2 + 4y + 4,$$

$$\therefore 2\sqrt{x} = y + 2,$$

$$\text{but } 2\sqrt{x} = \frac{2y}{\pm y - 1}$$

$$\therefore y + 2 = \frac{2y}{\pm y - 1}$$

$$\text{and first } y + 2 = \frac{2y}{y - 1}$$

$$\therefore y^2 + y - 2 = 2y.$$

$$y^2 - y + \frac{1}{4} = \frac{9}{4}.$$

$$y - \frac{1}{2} = \pm \frac{3}{2},$$

$$y = 2 \text{ or } -1.$$

$$\text{Again, } y + 2 = \frac{2y}{-y - 1},$$

$$\text{or } -y^2 - 3y - 2 = 2y,$$

$$y^2 + 5y + \frac{25}{4} = \frac{25}{4} - 2 = \frac{17}{4},$$

$$\therefore y + \frac{5}{2} = \pm \frac{\sqrt{17}}{2},$$

$$y = \frac{-5 \pm \sqrt{17}}{2}.$$

$$\therefore y = 2, \text{ or } -1, \text{ or } \frac{-5 \pm \sqrt{17}}{2},$$

$$\text{and } x = 4 \text{ or } \frac{1}{4} \text{ or } \frac{13 \pm \sqrt{17}}{8}.$$

32. Given $\dfrac{x + y + \sqrt{x^2 - y^2}}{x + y - \sqrt{x^2 - y^2}} = \dfrac{9}{8y}(x + y)$ and $(x^2 + y)^2 + x - y = 2x(x^2 + y) + 506$ } to find the values of x and y.

Dividing numerator and denominator of the left hand-side of the first equation by $\sqrt{x + y}$,

$$\frac{\sqrt{x + y} + \sqrt{x - y}}{\sqrt{x + y} - \sqrt{x - y}} = \frac{9}{8y}(x + y).$$

Again, multiplying numerator and denominator by

$$\sqrt{x+y} + \sqrt{x-y},$$

$$\frac{2x + 2\sqrt{x^2 - y^2}}{2y} = \frac{9}{8y}(x+y)$$

$$\text{or } (\sqrt{x+y} + \sqrt{x-y})^2 = \frac{9}{4}(x+y)$$

$$\text{and } \therefore \sqrt{x+y} + \sqrt{x-y} = \frac{3}{2}\sqrt{x+y},$$

$$\therefore \frac{\sqrt{x+y}}{2} = \sqrt{x-y}.$$

Squaring both sides, and clearing of fractions,

$$x + y = 4x - 4y$$

$$3x = 5y$$

$$y = \frac{3x}{5}.$$

Again, from the second equation,

$$(x^2 + y)^2 - 2x(x^2 + y) + x^2 = x^2 + y - x + 506,$$

$$(x^2 + y - x)^2 - (x^2 + y - x) + \frac{1}{4} = 506 + \frac{1}{4} = \frac{2025}{4}$$

$$x^2 + y - x = \frac{\pm 45 + 1}{2} = 23 \text{ or } -22,$$

$$\text{or } x^2 + \frac{3x}{5} - x = 23 \text{ or } -22,$$

$$x^2 - \frac{2x}{5} + \frac{1}{25} = 23 + \frac{1}{25} \quad \text{or } -22 + \frac{1}{25}$$

$$= \frac{576}{25} \text{ or } -\frac{549}{25},$$

$$x - \frac{1}{5} = \pm\frac{24}{5} \quad \text{or } \pm\frac{\sqrt{-549}}{5},$$

$$x = 5 \text{ or } -\frac{23}{5} \quad \text{or } \frac{1 \pm \sqrt{-549}}{5},$$

$$y = 3 \text{ or } -\frac{69}{25} \quad \text{or } \frac{3 \pm 3\sqrt{-549}}{25}.$$

33. Given $\frac{y}{x} \cdot \sqrt{\frac{x}{y}} + \frac{1}{2}\sqrt{\frac{x}{y}} \cdot \sqrt[4]{\frac{y^6}{x^3}} = 5$
and $\frac{2x^2}{y} - \frac{x}{3\sqrt{y}} = \frac{1}{3}$

to find the values of x and y.

From the first equation

$$\sqrt{\frac{y}{x}} + \frac{1}{2} \cdot \frac{x^{\frac{1}{2}}y^{\frac{3}{2}}}{y^{\frac{1}{2}}x^{\frac{3}{4}}} = 5,$$

$$\text{or } \frac{y^{\frac{1}{2}}}{x^{\frac{1}{2}}} + \frac{1}{2} \cdot \frac{y^{\frac{1}{4}}}{x^{\frac{1}{4}}} + \frac{1}{16} = \frac{1}{16} + 5 = \frac{81}{16},$$

$$\frac{y^{\frac{1}{4}}}{x^{\frac{1}{4}}} + \frac{1}{4} = \pm \frac{9}{4},$$

$$\frac{y^{\frac{1}{4}}}{x^{\frac{1}{4}}} = 2 \text{ or } -\frac{5}{2},$$

$$\frac{y}{x} = 16 \text{ or } \frac{625}{16}.$$

From the second equation,

$$\frac{x^2}{y} - \frac{1}{6} \cdot \frac{x}{\sqrt{y}} + \frac{1}{144} = \frac{1}{144} + \frac{1}{6} = \frac{25}{144}$$

$$\therefore \frac{x}{\sqrt{y}} - \frac{1}{12} = \pm \frac{5}{12},$$

$$\frac{x}{\sqrt{y}} = \frac{1}{2} \text{ or } -\frac{1}{3},$$

$$\text{and } \frac{x^2}{y} = \frac{1}{4} \text{ or } \frac{1}{9},$$

$$\therefore \frac{x^2}{y} \times \frac{y}{x} = \frac{1}{4} \times 16 \text{ or } \frac{1}{4} \times \frac{625}{16} \text{ or } \frac{1}{9} \times 16 \text{ or } \frac{1}{9} \times \frac{625}{16},$$

$$\therefore x = 4, \text{ or } \frac{625}{64}, \text{ or } \frac{16}{9}, \text{ or } \frac{625}{144},$$

$$\text{and } y = 64 \text{ or } \overline{\frac{625}{32}}^{\,2}, \text{ or } \frac{256}{9} \text{ or } \overline{\frac{-625}{48}}^{\,2}.$$

34. Given $\sqrt{\frac{x}{y}} + \sqrt{\frac{y}{x}} = \frac{61}{\sqrt{xy}} + 1$ and $\sqrt[4]{x^3y} + \sqrt[4]{y^3x} = 78$ } to find the values of x and y.

From the first equation,

$$\frac{x+y}{\sqrt{xy}} = \frac{61}{\sqrt{xy}} + 1,$$

$$\therefore x + y = 61 + \sqrt{xy}.$$

From the second,

$$\overline{xy}\big|^{\frac{1}{4}} (\sqrt{x} + \sqrt{y}) = 78,$$

$$\text{or } \sqrt{x} + \sqrt{y} = \frac{78}{\sqrt[4]{xy}},$$

$$\therefore x + y + 2\sqrt{xy} = \frac{6084}{\sqrt{xy}},$$

$$\text{and } \therefore x + y = \frac{6084}{\sqrt{xy}} - 2\sqrt{xy} = 61 + \sqrt{xy},$$

$$\therefore 6084 = 61\sqrt{xy} + 3xy,$$

$$\text{and } xy + \frac{61}{3}\sqrt{xy} + \overline{\frac{61}{6}}\Big|^2 = \frac{6084}{3} + \overline{\frac{61}{6}}\Big|^2 = \frac{76729}{36},$$

$$\therefore \sqrt{xy} + \frac{61}{6} = \pm\frac{277}{6}$$

$$\text{and } \sqrt{xy} = \frac{216}{6} \text{ or } -\frac{338}{6} = 36 \text{ or } -\frac{169}{3},$$

$$xy = 1296 \text{ or } \frac{28561}{9},$$

$$\text{but } x + y = 61 + \sqrt{xy} = 97,$$

$$\therefore x^2 + 2xy + y^2 = 9409 \text{ or } \frac{196}{9},$$

$$\text{but } 4xy = 5184 \text{ or } \frac{114244}{9},$$

$$x^2 - 2xy + y^2 = 4225 \text{ or } -\frac{114048}{9},$$

$$x - y = \pm 65 \text{ or } \pm 24\sqrt{-22}$$

$$\text{but } x + y = 97$$

$$x = 81, \text{ or } 16, \text{ or } \frac{97}{2} \pm 12\sqrt{-22},$$

$$y = 16 \text{ or } 81, \text{ or } \frac{97}{2} \mp 12\sqrt{-22}.$$

35. Given
$$\left.\begin{aligned} &\frac{y}{2x} + \frac{2}{3}\cdot\frac{y - \sqrt{x-1}}{y^2 - 2\sqrt{x^2-1}} = \frac{\sqrt{x+1}}{x} \\ &\text{and } \frac{1}{4}\cdot y^4 = y^2x - 1, \end{aligned}\right\}$$
to find the values of x and y.

From the second equation,

$$y^4 = 4y^2x - 4,$$

$$\therefore y^4 - 4y^2x + 4x^2 = 4(x^2 - 1),$$

$$\therefore y^2 - 2x = \pm 2\sqrt{x^2 - 1};$$

$$\therefore y^2 = 2x \pm 2\sqrt{x^2 - 1}$$

$$= (\sqrt{x+1} \pm \sqrt{x-1})^2$$

$$\text{and } y = \sqrt{x+1} \pm \sqrt{x-1}.$$

Substituting for $2\sqrt{x^2 - 1}$, $y^2 - 2x$ in the second equation,

$$\frac{y}{2x} + \frac{2}{3}\cdot\frac{y - \sqrt{x-1}}{2x} = \frac{\sqrt{x+1}}{x},$$

$$\text{or } \frac{y}{2} + \frac{1}{3}(y - \sqrt{x-1}) = \sqrt{x+1}.$$

Substituting for y, $\sqrt{x-1} + \sqrt{x-1}$

$$\frac{y}{2} + \frac{1}{3}\sqrt{x+1} = \sqrt{x+1},$$

$$\frac{y}{2} = \frac{2\sqrt{x+1}}{3} = \frac{\sqrt{x+1} + \sqrt{x-1}}{2};$$

$$\therefore \frac{\sqrt{x+1}}{3} = \sqrt{x-1}, \text{ and } x + 1 = 9x - 9;$$

$\therefore\ 8x = 10$ and $x = \frac{5}{4}$.

Again, substituting for y, $\sqrt{x+1} - \sqrt{x-1}$

$$\frac{y}{2} + \frac{1}{3}(\sqrt{x+1} - 2\sqrt{x-1}) = \sqrt{x+1}$$

$$\frac{y}{2} = \frac{2}{3}(\sqrt{x-1} + \sqrt{x+1}) = \frac{\sqrt{x+1} - \sqrt{x-1}}{2}$$

$$4\sqrt{x+1} + 4\sqrt{x-1} = 3\sqrt{x+1} - 3\sqrt{x-1}$$

$$\therefore\ \sqrt{x+1} = -7\sqrt{x-1}$$

$$x + 1 = 49x - 49,$$

$$48x = 50,$$

$$\text{and } x = \frac{50}{48} = \frac{25}{24};$$

$$\therefore\ x = \frac{5}{4} \text{ or } \frac{25}{24}.$$

Now $y^4 - 4y^2x^2 = -4$,

or $y^4 - 5y^2 = -4$,

$$y^4 - 5y^2 + \frac{25}{4} = \frac{9}{4},$$

$$y^2 - \frac{5}{2} = \pm\frac{3}{2},$$

$$y^2 = 4 \text{ or } 1,$$

$$y = \pm 2 \text{ or } \pm 1.$$

Again, $y^4 - \dfrac{25y^2}{6} = -4$,

$$y^4 - \frac{25y^2}{6} + \frac{625}{144} = \frac{625-576}{144} = \frac{49}{144};$$

$$\therefore\ y^2 - \frac{25}{12} = \pm\frac{7}{12},$$

$$y^2 = \frac{8}{3} \text{ or } \frac{3}{2};$$

$$\therefore\ y = \pm\frac{2}{3}\sqrt{6} \text{ or } \pm\frac{1}{2}\sqrt{6}.$$

36. Given $x^2 - y^2 = 3$,
and $(x^4 + y^4)^2 + x^2y^2(x^2 - y^2)^2 + x^2 - y^2 = 328$ } to find the values of x and y.

Substituting 3 for $x^2 - y^2$ in the second equation,

$$(x^4 + y^4)^2 + 9x^2y^2 + 3 = 328,$$

$$\text{and } x^4 - 2x^2y^2 + y^4 = 9;$$

$$\therefore x^4 + y^4 = 9 + 2x^2y^2;$$

$$\therefore \text{ also } (9 + 2x^2y^2)^2 + 9x^2y^2 = 325,$$

$$\text{or } 4x^4y^4 + 45\,x^2y^2 = 244;$$

$$\therefore x^4y^4 + \frac{45}{4}x^2y^2 + \overline{\frac{45}{8}\Big|}^2 = \overline{\frac{45}{8}\Big|}^2 + \frac{244}{4} = \frac{5929}{64};$$

$$\therefore x^2y^2 + \frac{45}{8} = \pm\frac{77}{8},$$

$$\text{and } x^2y^2 = 4 \text{ or } -\frac{61}{4}.$$

$$\begin{aligned} \text{Now } x^4 - 2x^2y^2 + y^4 &= 9, \\ 4x^2y^2 &= 16 \text{ or } -61. \\ \hline x^4 + 2x^2y^2 + y^4 &= 25 \text{ or } -52 \\ \text{and } x^2 + y^2 &= \pm 5 \text{ or } \pm 2\sqrt{-13}, \\ \text{but } x^2 - y^2 &= 3; \\ \hline \end{aligned}$$

$$\therefore x^2 = 4 \text{ or } -1 \text{ or } \frac{3 \pm 2\sqrt{-13}}{2},$$

$$\text{and } x = \pm 2 \text{ or } \pm\sqrt{-1} \text{ or } \pm\sqrt{\frac{3 \pm 2\sqrt{-13}}{2}}.$$

Similarly $y = \pm 1$ or $\pm 2\sqrt{-1}$ or $\pm\sqrt{\dfrac{3 \mp 2\sqrt{-13}}{2}}$.

37. Given $\dfrac{2y^2 - 8\sqrt{x}}{\sqrt{x}} + \sqrt{4y^2 - 16\sqrt{x}} = \dfrac{3\sqrt{x}}{2}$
and $\sqrt{x} + \sqrt{8.(y - \sqrt{x}) - 4} = y + 1$ }
to find the values of x and y.

From the first equation,

$$y^2 - 4\sqrt{x} + \sqrt{x}\sqrt{y^2-4\sqrt{x}} = \frac{3x}{4}$$

$$\therefore y^2 - 4\sqrt{x} + \sqrt{x}\sqrt{y^2-4\sqrt{x}} + \frac{x}{4} = \frac{3x}{4} + \frac{x}{4} = x;$$

$$\therefore \sqrt{y^2-4\sqrt{x}} + \frac{\sqrt{x}}{2} = \pm\sqrt{x},$$

$$\sqrt{y^2-4\sqrt{x}} = \frac{\sqrt{x}}{2} \text{ or } -\frac{3\sqrt{x}}{2}$$

$$y^2 - 4\sqrt{x} = \frac{x}{4} \text{ or } \frac{9x}{4},$$

$$\text{and } y^2 = \frac{x}{4} + 4\sqrt{x}, \text{ or } \frac{9x}{4} + 4\sqrt{x}.$$

Again, from the second equation,

$$y - \sqrt{x} + 1 = 2\sqrt{2(y-\sqrt{x}) - 1};$$

$$\therefore 2(y-\sqrt{x}) + 2 = 4\sqrt{2(y-\sqrt{x}) - 1},$$

$$2(y-\sqrt{x}) - 1 - 4\sqrt{2(y-\sqrt{x}) - 1} = -3;$$

$$\therefore (2(y-\sqrt{x}) - 1) - 4\sqrt{2(y-\sqrt{x}) - 1} + 4 = 1,$$

$$\sqrt{2(y-\sqrt{x}) - 1} - 2 = \pm 1,$$

$$\sqrt{2(y-\sqrt{x}) - 1} = 3 \text{ or } 1,$$

$$2(y-\sqrt{x}) - 1 = 9 \text{ or } 1.$$

$$\therefore y - \sqrt{x} = 5 \text{ or } 1,$$

$$y = 5 + \sqrt{x}, \text{ or } 1 + \sqrt{x};$$

$$\therefore y^2 = x + 10\sqrt{x} + 25, \text{ or } x + 2\sqrt{x} + 1,$$

$$\text{but } y^2 = \frac{x}{4} + 4\sqrt{x}, \text{ or } \frac{9x}{4} + 4\sqrt{x}.$$

First then,

$$x + 2\sqrt{x} + 1 = \frac{x}{4} + 4\sqrt{x} \text{ or } \frac{9x}{4} + 4\sqrt{x},$$

$$\text{or } x - 2\sqrt{x} + 1 = \frac{x}{4} \text{ or } \frac{9x}{4},$$

$$\sqrt{x} - 1 = \pm\frac{\sqrt{x}}{2} \text{ or } -\frac{3\sqrt{x}}{2};$$

$$\frac{\sqrt{x}}{2} = 1 \text{ or } \frac{3\sqrt{x}}{2} = 1 \text{ or } 5\sqrt{x} = 2;$$

$$\therefore\ \sqrt{x} = 2 \text{ or } \frac{2}{3} \text{ or } \sqrt{x} = \frac{2}{5};$$

$$\therefore\ x = 4 \text{ or } \frac{4}{9}, \text{ and } x = \frac{4}{25}.$$

Secondly,

$$x + 10\sqrt{x} + 25 = \frac{x}{4} + 4\sqrt{x};$$

$$\therefore\ \frac{3x}{4} + 6\sqrt{x} = -25,$$

$$x + 8\sqrt{x} + 16 = 16 - \frac{100}{3} = -\frac{52}{3},$$

$$\sqrt{x} + 4 = \pm\sqrt{-\frac{52}{3}} = \pm 2\sqrt{-\frac{13}{3}},$$

$$\sqrt{x} = -4 \pm 2\sqrt{-\frac{13}{3}},$$

$$\text{and } x = 16 \pm 16\sqrt{-\frac{13}{3}} - \frac{52}{3} = -\frac{4}{3} \mp 16\sqrt{-\frac{13}{3}}.$$

Thirdly,

$$x + 10\sqrt{x} + 25 = \frac{9x}{4} + 4\sqrt{x};$$

$$\therefore\ \frac{5x}{4} - 6\sqrt{x} = 25,$$

$$x - \frac{24\sqrt{x}}{5} = 20;$$

$$\therefore\ x - \frac{24\sqrt{x}}{5} + \frac{144}{25} = 20 + \frac{144}{25} = \frac{644}{25},$$

$$\sqrt{x} - \frac{12}{5} = \pm \frac{\sqrt{644}}{25},$$

$$\sqrt{x} = \frac{12 \pm \sqrt{644}}{25},$$

$$\text{and } x = \frac{788 \pm 24\sqrt{644}}{25};$$

$$\therefore x = 4, \text{ or } \frac{4}{9}; \text{ or } \frac{4}{25}, \text{ or } -\frac{4}{3} \mp 16\sqrt{-\frac{13}{3}},$$

$$\text{or } \frac{788 \pm 24\sqrt{644}}{25},$$

$$\text{and } y = 3, \text{ or } \frac{5}{3}; \text{ or } \frac{7}{5}, \text{ or } -1 \text{ or } 1 \pm 2\sqrt{-\frac{13}{3}}$$

$$\text{or } \frac{37 \pm \sqrt{644}}{5}.$$

THE END.

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LONDON:
Printed by William Clowes,
Stamford-street.